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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B. Tech

SEM: I- THEORY EXAMINATION (2022-2023)

Subject: Mathematical Foundations– I

Time: 3Hours **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A 20

1. Attempt all parts:-

1-a. If Eigen values of square matrix A is 2 , 3, 4 then Eigen values of A^{-1} are 1

(a) 2,3,4

(b) 2, -3, 4

(c) -2, -3, -4

(d) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ (CO1)

1-b. _____ is equal to the maximum number of linearly independent row vectors in matrix. 1

(a) Row matrix (b) Rank of matrix (c) Term matrix (d) Linear matrix (CO1)

1-c. A subset W is called subspace of vector space V(F) for a, b$\in$F and α, β$\in$V, if satisfy, 1

(a) aα - bβ $\in$ V

(b) aα × bβ $\in$ V

(c) aα ÷ bβ $\in$ V

(d) aα + bβ $\in$ V (CO2)

1-d. If S = $\{v_1, v_2, v_3, -----, v_n\}$ is a set of vectors in a finite dimensional vector space V , then S is called a basis for V if: 1

(a) S spans V

(b) S is linearly independent

(c) either (a) or (b) exists

(d) both (a) and (b) exists (CO2)

1-e. The curve $a^2y^2 = x^3(2a - x)$ is symmetrical about : 1

(a) $x = 0$

(b) $y = 0$

(c) $y = x$

(d) None of these (CO3)

1-f. If $y = (sin^{-1}x)^2$,then the value of $(1 - x^2)y_2 - xy_1$ is 1

(a) 1

(b) 2

(c) 2y

(d) 3 (CO3)

1-g. The Jacobian of p, q, r with respect to x , y , z is where p=x + y + z , q=y + z , r = z is 1

(a) 1

(b) 2

(c) -1

(d) 4 (CO4)

1-h. What is the saddle point? 1

a) Point where function has maximum value

b) Point where function has minimum value

c) Point where function has zero value

d) Point where function neither have maximum value nor minimum value (CO4)

1-i. If the price of an item is decreased by 10% and then increased by 10%, the net effect on the price of the item is: 1

(a) a decrease of 99%
(b) No change
(c) a decrease of 1%
(d) an increase of 1% (CO5)

1-j. If CHAIR is written as '12345', RENT is written as '5678', and then REAR is written as. 1

(a) 5635
(b) 5356
(c) 5365
(d) 5653 (CO5)

2. Attempt all parts:-

2.a. Find the value of P for which the rank of matrix $\begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is 1. (CO1) 2

2.b. Define dimension of linear transformation. (CO2) 2

2.c. If $z = \log\left(\frac{x^2}{y}\right)$, than find value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$. (CO3) 2

2.d. Expand $sinx$ in ascending powers of $(x - \frac{\pi}{2})$ up to second degree term. (CO4) 2

2.e. If BROWN is written as 'ZPMUL', then how is VIOLET written in that code language ? (CO5) 2

SECTION – B 30

3. Answer any <u>five</u> of the following-

3-a. Find the rank of matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing into Normal form. (CO1) 6

3-b. Find the Eigen values and Eigen vectors of the 3x3 matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. (CO1) 6

3-c. Consider the transformation $T : R^2 \to R^3$ defined by $\mathrm{T(x, y)} = (\mathrm{x}, 2\mathrm{x} - \mathrm{y}, 3\mathrm{x} + 4\mathrm{y})$. Show that T is linear transformation. (CO2) 6

3-d. If α and β are vectors in an inner product space then show that $\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$ (CO2) 6

3-e. Find n[th] derivative of $\frac{5x+12}{x^2+5x+6}$. (CO3) 6

3-f. Find the possible percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ if r_1, r_2, r_3 are each in error by +1.2%. (CO4) 6

3-g. 5 members of a team are weighted consecutively and their average weight calculated after each member is weighed. If average weight increases by one KG each time ,how much heavier is the last player than the first one? (CO5) 6

SECTION – C 50

4. Answer any <u>one</u> of the following-

4-a. Investigate the value of λ and μ so that the system of equations 2x +3y +5z = 9, 7x+3y-2z = 8, 2x +3y+λz = μ have (i) no solution (ii) a unique solution(iii) an infinite number of solutions . (CO1) 10

4-b. Verify Cayley Hamilton theorem for A=$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and hence find A^{-1}. (CO1) 10

5. Answer any one of the following-

5-a. Show that for the vectors $\alpha = (a_1, a_2)$ and $\beta = (b_1, b_2)$ from $V_2(F)$ the following defines an inner product space on $V_2(F)$:
$(\alpha, \beta) = 2a_1\,\overline{b_1} + a_1\,\overline{b_2} + a_2\,\overline{b_1} + a_2\,\overline{b_2}$. (CO2) 10

5-b. Let $T : R^4 \to R^3$ be a linear transformation defined by $T(x, y, z, w) = (x - y + z + w, x + 2z - w, x + y + 3z - 3w)$, then find Nullity of T and rank of T. (CO2) 10

6. Answer any one of the following-

6-a. If $y = \left[x + \sqrt{1 + x^2}\right]^m$, find $y_n(0)$. (CO3) 10

6-b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. (CO3) 10

7. Answer any one of the following-

7-a. Using Lagrange's method of multiplier , find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 1$. (CO4) 10

7-b. If u ,v, w are the roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (CO4) 10

8. Answer any one of the following-

8-a. (i) A toy is sold at 25% profit. If it had been sold at 15% loss, the selling price would have been Rs. 120 less. Find the cost of toy.
(ii) In an examination, 50% students failed in English and 40% in Mathematics and 15% students failed in both subjects. If 200students passed in both the subjects, find the number of students appeared in the exam. (CO5) 10

8-b. In a certain code, the following language is used: "Challenging world economy now" is written as "Sk Rk Tk Nk" ."Economy bad current issue" is written as "Pk Gk Sk Fk". "Current world looking good" is written as "Nk Mk Pk Vk". "Looking good challenging with" is written as "Vk Mk Rk Dk". The codes "Rk Mk Nk" may represent which of the following?
A) Challenging good bad
B) World economy bad
C) World looking challenging
D) None of these (CO5) 10