Printed page: 02 **Subject Code: AAS0104.**

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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY ,GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

(SEM: I - THEORY EXAMINATION(2020-2021)

Subject Name: Mathematical Foundations - I

Time : 3 Hours **Max. Marks : 100**

General Instructions:

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of 02 pages & 8 questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- Section A -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- Section B - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- Section C -Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10 marks each. You need to attempt any one part *a* or *b*.
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION – A

				CO
1.	**Answer all the parts-**		**[10×1=10]**	
	a.	The rank of matrix $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is equal to..... .	**(1)**	CO 1
	b.	If $A^2 + A - I = 0$, then A^{-1}=..... .	**(1)**	CO 1
	c.	Determine whether or not (1,1,1), (1,0,1) form a basis of R^3?	**(1)**	CO 2
	d.	State Rank-Nullity theorem.	**(1)**	CO 2
	e.	The nth derivative of Cos^2x is	**(1)**	CO 3
	f.	$Z = \frac{x^4+y^4}{x+y}$ is a homogeneous function of order	**(1)**	CO 3
	g.	If u = x (1-y), v = xy, then the value of the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ is..... .	**(1)**	CO 4
	h.	The stationary points of $f(x,y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1$ is	**(1)**	CO 4
	i.	If 16.5% of a quantity is 74.25, then total quantity is..... .	**(1)**	CO 5
	j.	The selling price of an article is Rs. 5400. The cost price of an article if loss is one-fifth of the selling price is (a) Rs. 6480 (b) Rs. 6500 (c) Rs. 7200 (d) Rs. 7600 .	**(1)**	CO 5
2.	**Answer all the parts-**		**[5×2=10]**	**CO**
	a.	The Eigen values of a matrix A are 2, 3, 1 then find the Eigen values of $A^2 + 3A - A^{-1}$.	**(2)**	CO 1
	b.	State True/false: x-axis is the asymptote to the curve $xy^2 = a^2(a - x)$.	**(2)**	CO 2
	c.	Explain basis and dimension of a vector space.	**(2)**	CO 3
	d.	Explain Lagrange's method of multipliers.	**(2)**	CO 4
	e.	Explain Coding with examples.	**(2)**	CO 5

SECTION – B

3. Answer any five of the following- **[5×6=30]** **CO**

a. Find inverse of the matrix by elementary transformations : $\begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ **(6)** **CO 1**

b. Let V be the set of ordered pairs (a, b) of real numbers with addition in V and scalar multiplication on V defined by (a ,b) + (c, d) = (a+c, b+d) and k (a, b) = (k a,0) . Is V a vector space? **(6)** **CO 2**

c. If $u = x\, f\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$. **(6)** **CO 3**

d. Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem for several variables. **(6)** **CO 4**

e. A number is first increased by 25% and then decreased by 25%. Find the net increase or decrease percent. **(6)** **CO 5**

f. Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. **(6)** **CO 4**

g. Trace the curve $x^3 + y^3 = 3axy$ (Folium of Descartes) . **(6)** **CO 3**

SECTION – C

4 Answer any one of the following- **[5×10=50]** **CO**

a. Solve the following system by Matrix method **(10)** **CO 1**

$$2x - 2y + 5z + 3w = 0$$
$$4x - y + z + w = 0$$
$$3x - 2y + 3z + 4w = 0$$
$$x - 3y + 7z + 6w = 0\ .$$

b. Diagonalize the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. **(10)** **CO 1**

5. Answer any one of the following-

a. Prove that if x and y are any two vectors in an inner product space, then $|(x, y)| \leq \|x\|\|y\|$. **(10)** **CO 2**

b. Show that the mapping $T: R^2 \to R^3$ defined as T(a, b) = (a – b, b – a, -a) is a linear transformation. Find the range, null-space and nullity of T. **(10)** **CO 2**

6. Answer any one of the following-

a. If $y = tan^{-1}x$, find $(y_n)_0$. **(10)** **CO 3**

b. If $u = f(y-z, z-x, x-y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(10)** **CO 3**

7. Answer any one of the following-

a. If $u = x + 2y + z, v = x - 2y + 3z \ and \ w = 2xy - xz + 4yz - 2z^2$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$ and find a relation between $u, v \ , and \ w$. **(10)** **CO 4**

b. In estimating the number of bricks in a pile which is measured to be (5m x10m x 5m) count of bricks is taken as 100 bricks per metre cube. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is Rs. 2,000 per thousand bricks. **(10)** **CO 4**

8. Answer any one of the following-

a. The mean wage of 100 workers working in a factory, running two shifts of 60 and 40 workers respectively is Rs. 38. The mean wage of 60 workers working in the morning shift is Rs.40. find the mean wage of 40 workers working in the evening shift. **(10)** **CO 5**

b. By selling an article for Rs. 240, a man makes profit of 20%, What is his C.P.? What would his profit percent be if he sold the article for Rs. 275?. **(10)** **CO 5**