Printed page:02 **Subject Code: AMIAS0103**

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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech.-(Master of Integrated Technology)

(SEM: FIRST SEMESTER THEORY EXAMINATION (2020-2021))

Subject Name: Engineering Mathematics-I

Time: 3 Hours **Max. Marks:100**

General Instructions:

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of 02 pages & 8 questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- Section A -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- Section B - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- Section C -Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10marks each. You need to attempt any one part *a or b.*
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION – A

1. Attempt all the parts. **[10×1=10]** **CO**

a. A is a singular matrix of order 3 with eigen values 2 and 3. The third eigen value is **(1)** **CO1**
(a) 1
(b) 0
(c) 4
(d) -1

b. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is **(1)** **CO1**
(a) 0
(b) 1
(c) 2
(d) 3

c. If $u = \frac{x^2}{a} + \frac{y^2}{b} - 7$ then $\frac{\partial u}{\partial x}$ is .... **(1)** **CO2**

d. If $u = x^2$ and $x = t^3$ then $\frac{du}{dt}$ is .... **(1)** **CO2**

e. If $x = rcos\emptyset$ and $y = rsin\emptyset$ then $\frac{\partial(x,y)}{\partial(r,\emptyset)}$ is .... **(1)** **CO3**

f. The function $z = y^2 + x^2y + x^4$ has a minimum at $(0,0)$. (T/F) **(1)** **CO3**

g. The value of the double integral $\int_{x=0}^{3}\int_{y=0}^{1}(x^2 + 3y^2)dydx$ is 12. (T/F) **(1)** **CO4**

h. The value of $\int_0^{\infty} e^{-x^2}dx$ is $\sqrt{\pi}$. (T/F) **(1)** **CO4**

i. The value of $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$ is .... **(1)** **CO5**

j. Insert the missing number: 11, 13, 17, 19, 23, 29, 31, 37, 41, (......). **(1)** **CO5**

2. Attempt all the parts. **[5×2=10]** **CO**

a. Find a and b such that $A = \begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as eigen values. **(2)** **CO1**

b. Find the n^{th} derivative of $y = sin\,(ax + b)$. **(2)** **CO2**

c. The radius of a sphere is found to be 10 meter with a possible error of 0.02 meter. What is the relative error in calculating the volume of sphere? **(2)** **CO3**

d. Prove that Beta function is symmetric. **(2)** **CO4**

e. If 50% of $(x - y)$ is 30% of $(x + y)$ then what percent of x is y? **(2)** **CO5**

SECTION – B

3.	**Answer any five of the following-**		**[5×6=30]**	**CO**
a.	Show that the system of equations $$\left.\begin{aligned}3x+4y+5z&=\alpha\\4x+5y+6z&=\beta\\5x+6y+7z&=\gamma\end{aligned}\right\}$$ is consistent only if α, β and γ are in arithmetic progression.		**(6)**	**CO1**
b.	Trace the curve $a^2y^2 = x^2(a^2 - x^2)$.		**(6)**	**CO2**
c.	Prove that $\frac{1}{(1-x)} = \frac{1}{3} + \frac{(x+2)}{3^2} + \frac{(x+2)^2}{3^3} + \frac{(x+2)^3}{3^4} + \cdots$....		**(6)**	**CO3**
d.	Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\, dydx$.		**(6)**	**CO4**
e.	Using the transformation $x + y = u$ and $y = uv$, show that $\int_0^1 \int_0^{1-x} e^{\left(\frac{y}{x+y}\right)} dydx = \frac{1}{2}(e-1)$.		**(6)**	**CO4**
f.	The selling price of 20 articles is equal to the cost price of 25 articles. Find the profit percent.		**(6)**	**CO5**
g.	If the word LEADER is coded as 20-13-9-12-13-26, how would you write LIGHT?		**(6)**	**CO5**

SECTION – C

4	**Answer any one of the following-**		**[5×10=50]**	**CO**
a.	State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}.		**(10)**	**CO1**
b.	Find eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.		**(10)**	**CO1**
5.	**Answer any one of the following-**			
a.	If $y = x^n logx$, prove that $(i)\ y_{n+1} = \frac{n!}{x}$ $\quad (ii)\ x^2y_{p+2} + (2p - 2n + 1)xy_{p+1} + (p-n)^2y_p = 0$.		**(10)**	**CO2**
b.	State and prove Euler's theorem for homogeneous function. Also prove that if $u = tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$.		**(10)**	**CO2**
6.	**Answer any one of the following-**			
a.	Expand x^y in powers of $(x-1)$ and $(y-1)$ upto the third degree terms.		**(10)**	**CO3**
b.	Find a point on the paraboloid $z = x^2 + y^2$ nearest to the point (3,-6,4).		**(10)**	**CO3**
7.	**Answer any one of the following-**			
a.	Prove by the method of double integration that the area lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.		**(10)**	**CO4**
b.	Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by using Dirichlet's theorem.		**(10)**	**CO4**
8.	**Answer any one of the following-**			
a.	A batsman makes a score of 87 runs in the 17^{th} inning and thus increases his average by 3. Find his average after 17^{th} inning.		**(10)**	**CO5**
b.	If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers.		**(10)**	**CO5**