## NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech.

SEM: I - CARRY OVER THEORY EXAMINATION - AUGUST 2022

Subject: Engineering Mathematics-I

Time: 3 Hours                                                                   Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.

2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.

3. Section B - Question No-3 is based on external choice carrying 6 marks each.

4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.

5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A                                                20

1. Attempt all parts:-

1-a.        If the rank of A is 2, then rank of $A'$ is  (CO1)                                         1

        (a) 3

        (b) 2

        (c) 8

        (d) 16

1-b.        If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of $A^{-1}$ are (CO1)        1

        (a) $4, 5^2, 7^3$

        (b) 4, 5, 7

        (c) 1/4, 1/5, 1/7

        (d) none of these

1-c.        If $u = \sin^{-1}\dfrac{x+2y+3z}{\sqrt{(x^8+y^8+z^8)}}$ then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$ is  (CO2)        1

        (a) 3 cot u

        (b) 3 tan u

        (c) -3 tan u

        (d) -3 cot u

1-d. The asymptotes parallel to the y-axis of the curve $x^2 y^2 - a^2 (x^2 + y^2) = 0$ is (CO2)  1

(a) 2a

(b) $x = a$

(c) $x = 3a$

(d) None of these

1-e. An error of 2% is made in measuring length and breadth then the percentage error in  1
the area of the rectangle is (CO3)

(a) 6

(b) 4

(c) 8

(d) 16

1-f. The expansion of sin x is (CO3)  1

(a) $1 - x - \dfrac{x^2}{2!} - \dfrac{x^3}{3!} - \dfrac{x^4}{4!} + \ldots\ldots$

(b) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} \ldots\ldots$

(c) $-x - \dfrac{x^3}{3!} - \dfrac{x^5}{5!} - \dfrac{x^7}{7!} \ldots\ldots$

(d) $x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dfrac{x^7}{7!} \ldots\ldots$

1-g. The value of $\Gamma n\, \Gamma(1-n)$ is (CO4)  1

(a) $\dfrac{\pi}{\sin n\pi}$

(b) $\dfrac{2\pi}{\sin n\pi}$

(c) $\dfrac{\pi}{\cos n\pi}$

(d) None of these

1-h. The Value of $\int_1^0 \int_0^1 (x + y)\, dx\, dy$ is (CO4)  1

(a) 2

(b) -1

(c) -2

(d) None of these

1-i. If two successive discount are 30% and 10%. Find single equivalent discount. (CO5)  1

(a) 35%

(b) 39%

(c) 41%

(d) None

1-j. If blue is coded as green, green is coded as white and white is code as black, and then what will be the code for the colour of grass? (CO5)　　1

(a) White

(b) Green

(c) Black

(d) None of These

2. Attempt all parts:-

2.a. Show that the following set of vectors (1, 1, 1, 1), (0, 1, 1, 1), (0, 0, 1, 1), (0, 0, 0, 1) are linearly independent. (CO1)　　2

2.b. Find the n$^{th}$ differential coefficients of $x^2\ e^x$. (CO2)　　2

2.c. In a sphere of radius r, if r is measured as 18.5 inches and a possible error of 0.1 inch then find possible error in surface area. (CO3)　　2

2.d. Evaluate the value of $\int_0^1 x^4(1-x)^3\,dx$. (CO4)　　2

2.e. Find the missing terms of 6, 9, 7, 10, 8, 11, ? (CO5)　　2

SECTION B　　30

3. Answer any <u>five</u> of the following:-

3-a. Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and verify Caley-Hamilton theorem　　6

for this matrix. Find A $^{-1}$ also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. (CO1)

3-b. Find the inverse of the matrix by using elementary transformations, where $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ (CO1)　　6

3-c. If $u = f(r, s, t)\ and\ r = \dfrac{x}{y}$ , $s = \dfrac{y}{z}$ , $t = \dfrac{z}{x}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. (CO2)　　6

3-d. If $u = \tan^{-1}\left(\dfrac{x3 + y3}{x - y}\right)$, prove that (CO2)　　6

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$ .

3.e. Expand $e^x \sin y$ in the powers of x and y in the neighborhood of $\left(0, \dfrac{\pi}{4}\right)$ up to the three degree terms. (CO3)   6

3.f. Evaluating by changing the order of integration $\displaystyle\int_0^1 \int_{2y}^2 e^{x^2} dx\, dy.$ (CO4)   6

3.g. In certain code language 'si po re' means 'book is thick', 'ti na re' means 'bag is heavy', 'ka si' means 'interesting book' and 'de ti' means 'that bag'. What should stand for 'that is interesting' in that code language? (CO5)   6

<center>SECTION C 50</center>

4. Answer any <u>one</u> of the following:-

4.a. Find the eigen values and eigen vectors of a matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (CO1)   10

4.b. Investigate for what values of $\lambda$ and $\mu$ the equations $x + 2y + z = 8,\ 2x + 2y + 2z = 13,\ 3x + 4y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) many solutions. (CO1)   10

5. Answer any <u>one</u> of the following:-

5.a. **If $y = \sin\left(a \sin^{-1} x\right)$, then find $y_n(0)$.** (CO2)   10

5.b. Trace the curve $y^2 (2a - x) = x^3$. (CO2)   10

6. Answer any <u>one</u> of the following:-

6.a. Show that the functions $u = x + y + z,\ v = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are functionally related. Find the relation between them. (CO3)   10

6.b. Find the maximum and minimum distances of the point (1, 2, -1) to the sphere $x^2 + y^2 + z^2 = 24$. (CO3)   10

7. Answer any <u>one</u> of the following:-

7.a. Show that $\beta(m, n) = \lceil m \lceil n / \lceil (m+n)$. (CO4)   10

7.b. Apply Dirichlet's integral to find the volume and mass contained in the first octant solid region of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ if the density at any point is $\rho\ (x,y,z) = kxyz$. (CO4)   10

8. Answer any <u>one</u> of the following:-

8.a.  (i) A candidate who gets 30% of the marks in a test fails by 50 marks. Another candidate   10
who get 320 marks fails by 30 marks. Find the maximum marks?

(ii)  Simplify:   $\dfrac{2\frac{3}{4}}{1\frac{5}{6}} \div \dfrac{7}{8} \times \left(\dfrac{1}{3} + \dfrac{1}{4}\right) + \dfrac{5}{7} \div \dfrac{3}{4} \text{ of } \dfrac{3}{7}$

(iii) If 'CARING' is coded as 'EDVGKC' and 'SHARES' is coded as 'UKEPBO' , how will
CASKET be coded as in the same code?  (CO5)

8.b.  (i) The average age of husband, wife and their child 3 years ago was 27 years and that of   10
wife and the child 5 years ago was 20 years. Find the present age of husband?

(ii) A tradesman sold an article at a loss of 20%. If the selling price had been increase by Rs.
100, there would have been a gain of 5%. Find the cost price of the article?

(iii) Find the missing term

122, 62, 32, ?, 9.5, 5.75 .  (CO5)