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Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech.

SEM: I - CARRY OVER THEORY EXAMINATION - AUGUST 2022

Subject: Mathematical Foundations-I

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. The value of λ for which the vectors (1, -2, λ), (2, -1, 5) and (3, -5, 7λ) are linearly dependent. (CO1) 1

(a) 1

(b) 5/14

(c) 0

(d) None of these

1-b. If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are. (CO1) 1

(a) 4, 5^2, 7^3

(b) 4,5,7

(c) 1/4, 1/5, 1/7

(d) none of these

1-c. A subset W is called subspace of vector space V(F) for $a, b \in F$ and $\alpha, \beta \in V$, if satisfy (CO2) 1

(a) $a\alpha - b\beta \in V$

(b) $a\alpha\ x\ b\beta \in V$

(c) $a\alpha \div b\beta \in V$

(d) $a\alpha + b\beta \in V$

1-d. If T : $R^2 \rightarrow R^3$ is a linear transformation T(1, 0) = (2, 3, 1) and T(1, 1) = (3, 0, 2) then which of the following statement is correct ? (CO2) 1

(a) T(x y) = (x +y, 2x + y, 3x -3y)

(b) T(x y) = (2x +y, 3x -3y, x + y,)

(c) T(x y) = (2x -y, 3x +3y, x - y,)

(d) T(x y) = (x -y, 2x -y, 3x + 3y,)

1-e. If the two tangents are real and coincide then origin is (CO3) 1

(a) Node

(b) cusp

(c) Both node and cusp

(d) None of these

1-f. If $u = \frac{x^n - 1}{x - 1}$ then the value of y_n is (CO3) 1

(a) nx^n

(b) $n!$

(c) $(n+1)!$

(d) 0

1-g. Percentage error in the area of a rectangle when an error of +1 percent is made in measuring its length and breadth is given by (CO4) 1

(a) 4%

(b) 5%

(c) 2%

(d) 6%

1-h. The point at which the function $x^3 - 4xy + 2y^2$, maximum or minimum is. (CO4) 1

(a) $\left(\frac{4}{3}, \frac{2}{3}\right)$

(b) $\left(\frac{4}{3}, \frac{1}{3}\right)$

(c) $\left(1, \frac{1}{3}\right)$

(d) $\left(\frac{4}{3}, \frac{4}{3}\right)$

1-i. The average of all prime numbers between 30 and 50 is (CO5) 1

(a) 38.8

(b) 39.8

(c) 40.8

(d) None of these

1-j. If blue is coded as green, green is coded as white and white is code as black, and then what will be the code for the colour of grass? (CO5) 1

(a) White

(b) Green

(c) Black

(d) None of These

2. Attempt all parts:-

2.a. Show that the matrix $\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary matrix if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ (CO1). 2

2.b. Show that the three vectors (1, 1, -1), (2, -3, 5) and (-2, 1, 4) of R^3 are linearly independent. (CO2) 2

2.c. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$, *Show that* $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. (CO3) 2

2.d. If $u = x + 2y + z,\ v = x + 2y + 3z,\ w = 2x + 3y + 5z$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (CO4) 2

2.e. A man earns Rs. 4000 in a month and saves 30% of his income. What is his expenditures in a year? (CO5) 2

SECTION B 30

3. Answer any <u>five</u> of the following:-

3-a. Find the inverse of the matrix by using elementary transformations, where $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$.(CO1) 6

3-b. Find all values of μ for which rank of the matrix $\begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is equal to 3. (CO1) 6

3-c. Show that the vectors (1, 2, 1), (2, 1, 0), (1, -1, 2) form a basis of R^3. (CO2) 6

3-d. If α and β are vectors in an inner product space then show that $\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$. (CO2) 6

3.e. If $x^x y^y z^z = c$ then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$. (CO3) 6

3.f. Expand $e^x \cos y$ about the point (0,0) up to the three degree terms. (CO4) 6

3.g. In an examination, 34% of the students failed in Mathematics and 42% failed in English. If 20% of students failed in both the subjects, then the percentage of students who passed was. (CO5) 6

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence , compute A $^{-1}$. Also find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$ (CO1) 10

4-b. Determine the value of λ and μ that the equations $\begin{aligned} 2x + 3y + 5z &= 9 \\ 7x + 3y - 2z &= 8 \\ 2x + 3y + \lambda z &= \mu \end{aligned}$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.(CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Let $v = R^2 = \{(x, y): x, y \in R\}$ *and* F = R. Define the addition and scalar multiplication in R ^{2} as follows $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ *and* $a(x, y) = (ax, ay)$. Show that R ^{2} is a vector space over R. (CO2) 10

5-b. Show that the set W of the elements of the vector space V $_3$(R) of the form $(x + 2y, y, -x + 3y)$ *where* $x, y \in R$ *is a subspace of* $V_3(R)$. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. Trace the curve $y^2(2a - x) = x^3$ (CO3) 10

6-b. If $y = \sin(a \sin^{-1} x)$, then find $y_n(0)$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. In estimating the number of bricks in a pile which is measured to be (5 m x 10 m x 5 m) the count of bricks is taken as 100 bricks/meter ^{3}. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is Rs. 2000 per thousand bricks. (CO4) 10

7-b. If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) In certain code language SERIES is coded as 5625 and PIPE is coded as 2116. How will WAP be coded in the same code language?(CO5) 10

(ii) The average marks obtained by 22 candidates in an examination are 45. The average marks of the first 10 candidates are 55 and those of the last eleven are 40.The number of marks obtained by the eleventh candidate is ?(CO5)

(iii) A candidate scores 25 % marks and fails by 30 marks, while another candidate who scores 50 % marks get 20 marks more than the minimum marks required to pass the examinations. Find the maximum marks for the examination.(CO5)

8-b. (i) If the price of an item is decreased by 10% and then increased by 10%, the net effect on the price of the item is:(CO5) 10

(ii) The average marks obtained by 40 students of a class is 86. If the 5 highest marks are removed, the average reduced by one marks. The average marks of the top 5 students is ?(CO5)

(iii) Find the missing terms: 1, 2, 6, 7, 21, 22, 66, 67, ? (CO5)