Printed Page:- 05 **Subject Code:- AAS0103**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - CARRY OVER THEORY EXAMINATION - AUGUST 2023

Subject: Engineering mathematics I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. Inverse of unitary matrix is a (CO1) 1

(a) symmetric matrix

(b) skew-symmetric matrix

(c) unitary matrix

(d) None of these

1-b. If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are (CO1) 1

(a) 4, 5^2, 7^3

(b) 4, 5, 7

(c) 1/4, 1/5, 1/7

(d) none of these

1-c. The nth derivative of cos(ax+b) is (CO2) 1

(a) $a^n \cos(ax+b)$

(b) $a^n \cos\left(ax+b+\frac{n\pi}{2}\right)$

(c) $a^n \cos\left(ax+b+\frac{n\pi}{4}\right)$

(d) None of these

1-d. **If $u = x^3 + y^3$, where $x = a\cos t$, $y = b\sin t$, then the value of $\frac{du}{dt}$ is** (CO2) 1

(a) $-3a^3\cos^2 t\sin t + 3b^3\sin^2 t\cos t$

(b) $3a^3\cos^2 t\sin t - 3b^3\sin^2 t\cos t$

(c) $3a^3\cos^2 t\sin t + 3b^3\sin^2 t\cos t$

(d) $3a^3\cos^2 t\sin t + 3b^3\cos^2 t\cos t$

1-e. Maclaurin's series for f(x) is (CO3) 1

(a) $f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\ldots\ldots.$

(b) $f(x)+\frac{x}{1!}f'(x)+\frac{x^2}{2!}f''(x)+\frac{x^3}{3!}f'''(x)+\ldots\ldots.$

(c) $f(0)+\frac{x}{1}f'(0)+\frac{x^2}{2}f''(0)+\frac{x^3}{3}f'''(0)+\ldots\ldots.$

(d) $f(x)+\frac{1}{1!}f'(x)+\frac{1}{2!}f''(x)+\frac{1}{3!}f'''(x)+\ldots\ldots.$

1-f. If u and v are the function of x and y then value of $\frac{\partial(u,v)}{\partial(x,y)}\cdot\frac{\partial(x,y)}{\partial(u,v)}$ is (CO3) 1

(a) 1

(b) 0

(c) x.y

(d) u.v

1-g. Integral value of $\int_0^\infty x^2 e^{-x}\,dx$ is (CO4) 1

(a) -1

(b) 1

(c) 2

(d) None of these

1-h. The value of Γ(5/2) is (CO4) 1

(a) $2\sqrt{\pi}$

(b) $\frac{4}{3}\sqrt{\pi}$

(c) $\frac{3}{4}\sqrt{\pi}$

(d) None of these

1-i. In the college election, a candidate secure 62% of the votes and elected by a majority of 144 votes. The total number of votes polled is (CO5) 1

(a) 800

(b) 925

(c) 120

(d) 600

1-j. If two successive discount are 30% and 10%. Find single equivalent discount. (CO5) 1

(a) 35%

(b) 39%

(c) 41%

(d) None

2. Attempt all parts:-

2.a. Find the value of P for which the matrix $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is of rank 1. (CO1) 2

2.b. Find the n^{th} differential coefficients of $x^2 e^x$. (CO2) 2

2.c. If $x = u + v, y = uv$ then Find $\frac{\partial(u,v)}{\partial(x,y)}$. (CO3) 2

2.d. Calculate the volume of the solid bounded by x = 0, y = 0, z = 0 and x + y + z = 1. (CO4) 2

2.e. If out of 10 selected students for an examination, 3 were of 20 years age, 4 of 21 years and 3 of 22 years, then the average age of the group ? (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. Test the consistency of system of equation $10y + 3z = 0,\ 3x + 3y + z = 0,\ 2x - 3y - z = 5,\ x + 2y = 4$. (CO1) 6

3-b. Find the rank of matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 1 & 2 & -8 \end{bmatrix}$. (CO1) 6

3-c. If $u = f(r, s, t)$ and $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$ (CO2) 6

3-d. Trace the following curve $y^2 (a - x) = x^2 (a + x)$. (CO2) 6

3-e. Obtain the series for $\ln(1+x)$ then find the expansion of $\ln\left(\frac{1+x}{1-x}\right)$ and hence 6 find the value of $\ln\frac{11}{9}$. (CO3)

3.f. Evaluating by changing the order of integration $\int_0^1\int_{2y}^2 e^{x^2}dxdy$. (CO4) 6

3.g. In certain code language 'si po re' means 'book is thick', 'ti na re' means 'bag is heavy', 'ka si' means 'interesting book' and 'de ti' means 'that bag'. What should stand for 'that is interesting' in that code language? (CO5) 6

SECTION C **50**

4. Answer any <u>one</u> of the following:-

4-a. Verify Caley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence compute A $^{-1}$. Also evaluate $A^6-6A^5+9A^4-2A^3-12A^2+23A-9I$. (CO1) . 10

4-b. Find the eigen values and eigen vectotrs of a matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. If $y = \sin^{-1}x$, then prove that $(1-x^2)\,y_2 - xy_1 = 0$ and $(1-x^2)\,y_{n+2}-(2n+1)\,xy_{n+1}-n^2y_n = 0$. (CO2) 10

5-b. If $y = e^{a\sin^{-1}x}$, then prove that $(1-x^2)\,y_{n+2}-(2n+1)\,xy_{n+1}-(n^2+a^2)y_n = 0$. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. If $u=\frac{x+y}{z}, v=\frac{y+z}{x}, w=\frac{y(x+y+z)}{xz}$, then show that u, v, w are not independent and find the relation between them. (CO3) 10

6-b. Use the method of Lagrange's multiplier to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Apply Dirichlet's integral to find the volume and mass contained in the first octant solid region of the curve $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, if the density at any point is ρ (x,y,z) = kxyz. (CO4) 10

7-b. Evaluate $\iint_R (x+y)^2dxdy$ where R is the parallelogram in xy plane with vertices 10

(1, 0), (3, 1), (2, 2) and (0, 1) using the transformation u = x+y and v = x - 2y. (CO4)

8. Answer any <u>one</u> of the following:-

8-a. (i) A shopkeeper allows a 10% discount of to his customers and still gains 20%. Find the marked price of the article which costs Rs 450. 10

(ii) The average of marks of 17 students in an examination was calculated as 71. But it was later found that the mark of one student had been wrongly entered as 65 instead of 56 and another as 24 instead of 50. Find the correct average?

(iii) If the numerator of a fraction is increased by 20% and its denominator is decreased by 10%, the fraction becomes 3/2. Find the original fraction. (CO5)

8-b. (i) If the price of an item is decreased by 10% and then increased by 10%, then what is the net effect on the price of the item? 10

(ii) The average marks obtained by 40 students of a class is 86. If the 5 highest marks are removed and the average reduced by one mark. Find the average marks of the top 5 students?

(iii) Find the missing terms: 1, 2, 6, 7, 21, 22, 66, 67, ?
(CO5)