Printed Page:- 5 **Subject Code:- AAS0104**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - CARRY OVER THEORY EXAMINATION - AUGUST 2023

Subject: Mathematical Foundations-I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. If $A=\begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then the eigen values of A^2 are (CO1) 1

(a) 1,7,9

(b) 1,4,9

(c) 1,1,2

(d) 3,6,9

1-b. If the rank of A is 2, then rank of A' is (CO1) 1

(a) 3

(b) 2

(c) 8

(d) 16

1-c. Which condition exist for function to be linear transformation ? (CO2) 1

(a) $T(a\alpha+b\beta)=aT(\beta)+bT(\alpha)$

(b) $T(a\alpha+b\beta)=aT(\alpha)+bT(\alpha)$

(c) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$

(d) None of these

1-d. In the vector space V if $a + b = b + a$; where $a, b \in V$ is called (CO2) 1

(a) Associativity

(b) Additive inverse

(c) Commutative

(d) None of these

1-e. If $z = u^2 + v^2, u = r\cos\theta, v = r\sin\theta$, then the value of $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ are (CO3) 1

(a) $1, 2$

(b) $2r, 5$

(c) $2r, 0$

(d) $0, 0$

1-f. The nth derivative of cos(ax+b) is (CO 3) 1

(a) $a^n \cos(ax + b)$

(b) $a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$

(c) $a^n \cos\left(ax + b + \frac{n\pi}{4}\right)$

(d) None of these

1-g. If $u = x + y + z,\ v = 2x + 2y + 2z,\ w = \frac{1}{2}(x + y + z)$ are functionally dependent then the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ is (CO4) 1

(a) 1

(b) 2

(c) 3

(d) 0

1-h. Percentage error in the area of a rectangle when an error of +1 percent is made in measuring its length and breadth is given by (CO4) 1

(a) 4%

(b) 5%

(c) 2%

(d) 6%

1-i. A man had 7 children. When their average age was 12 years, a child aged 6 1

years died. The average age of remaining six children is (CO5)

(a) 13 years

(b) 10 years

(c) 11 years

(d) 14 years

1-j. If 35+48=40, 23+34=24, 15+25=26, then 11+21=? (CO5) 1

(a) 15

(b) 18

(c) 20

(d) 10

2. Attempt all parts:-

2.a. Show that $A=\begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is skew-Hermitian matrix. (CO1) 2

2.b. The subset S { (1, 0, 0), (0, 1, 0), (0, 0, 1) } of the vector space R^3 is linearly..... (CO2) 2

2.c. State the Leibnitz theorem. (CO3) 2

2.d. Determine the points where the function $u = x^2+y^2+6x+12$ has a maximum or minimum. (CO4) 2

2.e. Find the missing terms of 6, 5, 7, 12.5, 27, ? (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. Find the rank of matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 1 & 2 & -8 \end{bmatrix}$ (CO1) 6

3-b. Show that the equations $x-4y+7z=14,\ 3x+8y-2z=13,\ 7x-8y+26z=5$ are not consistent. (CO1) 6

3-c. Determine whether or not the following vectors form a basis of R^3 : (1, 2, 1), (1, 2, 5), (5, 3, 4). (CO2) 6

3-d. Prove that the set S = { (1, 3, -1), 2, 7, -3), (4, 8, -7) } spans R^3 . (CO2) 6

3.e. If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial}{\partial}=-3\tan u$. (CO3) 6

3.f. Expand $e^x \sin y$ in the powers of x and y in the neighborhood of $\left(0, \frac{\pi}{4}\right)$ up to the three degree terms. (CO4) 6

3.g. A machine is sold for Rs5060 at a gain of 10%. What would have been the gain or loss % if it had been sold for Rs 4370? (CO5) 6

SECTION C **50**

4. Answer any <u>one</u> of the following:-

4-a. Find the eigen values and eigen vectotrs of a matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (CO1) 10

4-b. Show that the system of equations $3x+4y+5z=a, 4x+5y+6z=b, 5x+6y+7z=c$, does not have solution unless $a+c=2b$. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Show that the set of vectors of the form $\{2a-3b, a-5c, a, 4c-b, c\}$ in R^5 form a subspace of R^5 under the usual operations. (CO2) 10

5-b. Show that the mapping $T: R^2 \to R^3$ defined as $T(a,b)=(a-b, b-a, -a)$ is a linear transformation. Find the range, null-space and nullity of T. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, show that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2 = 4u$. (CO3) 10

6-b. If $y = \sin(a\sin^{-1}x)$, then find $y_n(0)$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. A rectangular box closed at the top is of given volume, what must be the dimensions so that the surface area is minimum. (CO4) 10

7-b. If $u^3+v^3+w^3=x+y+z$, $u^2+v^2+w^2=x^3+y^3+z^3$ and $u+v+w=x^2+y^2+z^2$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) In certain code language SERIES is coded as 5625 and PIPE is coded as 2116. How will WAP be coded in the same code language? 10

(ii) The average marks obtained by 22 candidates in an examination are 45. The average marks of the first 10 candidates are 55 and those of the last eleven are 40.The number of marks obtained by the eleventh candidate is ?

(iii) A candidate scores 25 % marks and fails by 30 marks, while another candidate who scores 50 % marks get 20 marks more than the minimum marks required to pass the examinations. Find the maximum marks for the examination. (CO5)

8-b. (i) An article is listed at Rs. 1800 and two successive discounts of 8% and 8% are given on it. How much would the seller gain or loss, if he gives a single discount of 16% instead of two discounts? 10

(ii) In certain code, RELATION is written as ZKDQMNHS and NOSE is written as NMDR. How will MISTER be written in that code?

(iii) Out of four numbers the average of the first three is 16 and that of the last three is 15. If the last number is 20 then the find the first number. (CO5)