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**NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA**

**(An Autonomous Institute Affiliated to AKTU, Lucknow)**

**B.Tech**

SEM:II CARRY OVER THEORY EXAMINATION-AUGUST 2023

**Subject: Mathematical Foundations - II**

**Time: 3 Hours**

**Max. Marks: 100**

**General Instructions:**

**IMP:** Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C**. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

**SECTION A**

**20**

**1. Attempt all parts:-**

- 1-a. Evaluate the integral  $\int_0^3 \int_1^2 xy(1+x+y) dy dx$  (CO1) 1
- (a) 122/3
- (b) 120/4
- (c) 123/4
- (d) None of these
- 1-b. Evaluate  $\sqrt{-\frac{3}{2}}$  (CO1) 1
- (a)  $\frac{\sqrt{\pi}}{3}$
- (b)  $\frac{4\sqrt{\pi}}{3}$
- (c)  $-\frac{3\sqrt{\pi}}{4}$
- (d) None of these

- 1-c. Find the general solution of the second order linear differential equation  $(d^2y/dx^2) - 3(dy/dx) + 2y = 0$ . (CO2) 1
- (a)  $y = Ae^x + Be^{2x}$   
 (b)  $y = Ae^x + Be^{-2x}$   
 (c)  $y = Ax + 2Bx$   
 (d)  $y = A + Bx$
- 1-d. Find the Particular Integral of  $(D^2 - 1)y = x^2$  (CO 2) 1
- (a)  $-(x^2 + 2)$   
 (b)  $-x^2$   
 (c)  $(x + 2)$   
 (d)  $(x^2 + 2)$
- 1-e. The Auxiliary equation of the partial differential equation  $(x^2 - y^2 - z^2)p + 2xyq = 2xz$  is (CO3) 1
- (a)  $\frac{dx}{x^2 + y^2 + z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$   
 (b)  $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xz} = \frac{dz}{2xy}$   
 (c)  $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$   
 (d) None of these
- 1-f. The Complementary function of partial differential equation  $(D - D' - 1)(D - D' - 3)z = 0$  is (CO3) 1
- (a)  $C.F. = e^x f_1(y+x) + e^{-3x} f_2(y+x)$   
 (b)  $C.F. = e^x f_1(y+x) + e^{-3x} f_2(y-x)$   
 (c)  $C.F. = e^x f_1(y+x) + e^{3x} f_2(y+x)$   
 (d)  $C.F. = e^x f_1(y-x) + e^{3x} f_2(y+x)$
- 1-g. Inverse Laplace of the function  $f(s) = \frac{e^{-s}}{s}$  is (CO 4) 1
- (a)  $u(t-1)$   
 (b)  $u(t+1)$   
 (c)  $-u(t+1)$   
 (d) None of these
- 1-h. Laplace transform of  $te^{-t}$  is (CO 4) 1
- (a)  $\frac{2}{(s+1)^2}$

$$(b) \frac{1}{(s+1)^2}$$

$$(c) \frac{1}{(s-1)^2}$$

(d) None of these

1-i. Radha moves towards South-east a distance of 7 m, and then she moves towards West and travels a distance of 14 m. From here, she moves towards North-west a distance of 7 m and finally she moves a distance of 4 m towards East and stood at that point. How far is the starting point from where she stood? (CO5) 1

(a) 3 m

(b) 4 m

(c) 11 m

(d) 10 m

1-j. If  $P : Q = 2 : 3$ ,  $Q : R = 4 : 5$  and  $R : S = 6 : 7$ , then  $P : S = ?$  (CO5) 1

(a) 18:25

(b) 17:25

(c) 16:35

(d) 8:11

## 2. Attempt all parts:-

2.a. Find the area lying between  $y=x$ ,  $x=0$  and  $y=0$ . (CO1) 2

2.b. Write the relation between P and Q if  $e^{-x}$  is a part of the C.F. of the linear differential equation  $(d^2y/dx^2) + P(dy/dx) + Qy = R$  (CO2) 2

2.c. Find the Particular Integral of  $D(D - 2D' - 3)z = e^{x+2y}$ . (CO3) 2

2.d. Find Laplace transform of the function  $F(t) = \frac{\cos at - \cos bt}{t}$ . (CO4) 2

2.e. After 8 years, Anil will be three times as he was 8 years ago. what is his present age? (CO5) 2

## SECTION B

30

### 3. Answer any five of the following:-

3-a. Evaluate the integral by changing the order of integration  $\int_0^2 \int_{\frac{x^2}{4}}^{3-x} x y dy dx$  6  
(CO1)

3-b. Prove that  $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$  where  $m > 0$ ,  $n > 0$ . (CO1) 6

- 3-c. Find the solution of the differential equation  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$ . (CO2) 6
- 3-d. Solve the differential equation by the method of variation of parameters  $\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$ . (CO2) 6
- 3.e. Solve the linear partial differential equation  $4r + 2s + t = 2(y - x) + \sin(x - y)$ . (CO3) 6
- 3.f. Find the inverse Laplace Transform of the function  $f(s) = \left[ \log\left(\frac{s^2 + 4s + 5}{s^2 + 2s + 5}\right) \right]$ . (CO4) 6
- 3.g. (i) A girl leaves from her home. She first walks 30 m in North-West direction and then 30 m in South-West direction. Next, she walks 30 m in South-East direction. Finally, she turns towards her house. In which direction is she moving? (ii) Kashish goes 30m North, then turns right and walks 40 m, then again turns right and walks 20 m, then again turns right and walks 40 m. How far is he from his original position? (CO5) 6

### SECTION C

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#### 4. Answer any one of the following:-

- 4-a. Evaluate  $\iint_R (x+y)^2 dx dy$ , where R is the parallelogram in the xy-plane with vertices (1,0), (3,1), (2,2), (0,1), using the transformation  $u=x+y$  and  $v=x-2y$ . (CO1) 10
- 4-b. The plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  meets the axes in A, B and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. Also find its mass if the density at any point is  $kxyz$ . (CO1) 10

#### 5. Answer any one of the following:-

- 5-a. Solve the system of simultaneous differential equations  $\frac{dx}{dt} + 5x + y = e^t$ ,  $\frac{dy}{dt} + x + 5y = e^{5t}$  (CO2) 10
- 5-b. Solve the following differential equation by changing to independent variable  $x \frac{d^2y}{dx^2} + (4x^2 - 1) \frac{dy}{dx} + 4x^3y = 2x^3$ . (CO2) 10

#### 6. Answer any one of the following:-

6-a. Solve:  $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)$  (CO3) 10

6-b. Solve the linear partial differential equation 10

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y. \quad (\text{CO3})$$

**7. Answer any one of the following:-**

7-a. Solve the following differential equation by using Laplace transform 10  
 $y'' + 3y' + 2y = te^{-t}$ , where  $y(0) = 1, y'(0) = 0$ . (CO4)

7-b. Apply convolution theorem to evaluate:  $L^{-1} \left[ \frac{s}{(s^2 + 4)^2} \right]$ . (CO4) 10

**8. Answer any one of the following:-**

8-a. (i) If Rs.7500 are borrowed at C.I at the rate of 4% per annum, find the amount 10  
to be paid after 2 years?

(ii) The difference between simple interest and compound interest on a sum for 2 years at 8%, when the interest is compounded annually Rs. 16. What is the difference in two interests, if the interest was compounded half-yearly? (CO5)

8-b. (i) In an alloy, the ratio of copper and zinc is 5:2. If 1.250 kg of zinc is mixed in 10  
17 kg 500 g alloy, what is the ratio of copper and zinc?

(ii) 40 L of a mixture of milk and water contains 10 % water. How much water must be added to make the water 20 % in the new mixture?(CO5)