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Printed Page:- 05 **Subject Code:- ACSBS0205**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM:II CARRY OVER THEORY EXAMINATION- AUGUST 2023

Subject: Linear Algebra

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. Find the adjoint of a matrix $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$. (CO1) 1

(a) $\begin{bmatrix}5 & 10\\ 3 & 6\end{bmatrix}$

(b) $\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$

(c) $\begin{bmatrix}-4 & -2\\ -3 & -1\end{bmatrix}$

(d) None of these

1-b. If rows and columns of the determinants are interchanged, then its value (CO1) 1

(a) remains unchanged

(b) becomes change

(c) its doubled

(d) none of these

1-c. The rank of matrix $A = \begin{bmatrix} 5 & 10 \\ 3 & 6 \end{bmatrix}$ is (CO2) 1

(a) 6

(b) 5

(c) 1

(d) none of these

1-d. If the system of equations x+2y-3z=1, (p+2)z=3, (2p+1)y+z=2 is inconsistent then the value of p is (CO2) 1

(a) -2

(b) -1/2

(c) 0

(d) none of these

1-e. The null space of linear transformation from R^3 into R^3 defined as (CO3) 1

(a) (1, 2, 3)

(b) (1, 0, 0)

(c) (0, 1, 0)

(d) (0, 0, 0)

1-f. If α and β are orthogonal unit vectors then distance between α and β is ? (CO3) 1

(a) 1

(b) 0

(c) $\sqrt{2}$

(d) 2

1-g. A square matrix A is positive if A is symmetric matrix and all the eigenvalues are (CO4) 1

(a) Positive

(b) Negative

(c) Imaginary

(d) None of these

1-h. If A is an unitary matrix, then $|A|$ is CO 4 1

(a) 1

(b) -1

(c) ± 1

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(d) None of these

1-i. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ then the Eigen value of AA^T are (CO5) 1

(a) 9,9

(b) 1,1

(c) 1,9

(d) None of these

1-j. If $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$ then square root of Eigen value of A^TA are (CO5) 1

(a) $\sqrt{45}$,$\sqrt{5}$

(b) $\sqrt{5}$,$\sqrt{5}$

(c) $\sqrt{40}$,$\sqrt{5}$

(d) None of these

2. Attempt all parts:-

2.a. Express $A = \begin{bmatrix} 5 & 2 \\ 3 & -6 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrix. (CO1) 2

2.b. If the sytem of equations $\lambda x + y + z = 0,\ -x + \lambda y + z = 0,\ -x - y + \lambda z = o$ has a non zero solution, then find the possible value of λ. (CO2) 2

2.c. In an inner product space V(F), prove that $(a\alpha + b\beta, \gamma) = a(\alpha,\gamma) - b(\beta,\gamma)$. (CO3) 2

2.d. If A is Hermitian matrix then prove that iA is skew Hermitian matrix. 2

2.e. If $A = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$ then find the square root of the Eigen value of A. (CO5) 2

SECTION B 30

3. Answer any <u>five</u> of the following:-

3-a. Find the inverse of the matrix by using E – transformation, where $A = \begin{bmatrix} i & -1 & 2i \\ 2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$. (CO1) 6

3-b. Solve by Cramer's rule: x+y+z =2, 2x+y+3z=9 and x-3y+z=10 (CO1) 6

3-c. Solve by LU decomposition method : 3x+y+z=4, x+2y+2z=3,2x+y+3z=4. (CO2) 6

3-d. Solve the homogeneous system of equations: (CO2) 6

$x_1 + 3x_2 + 2x_3 = 0,$

$2x_1 - x_2 + 3x_3 = 0,$

$3x_1 - 5x_2 + 4x_3 = 0,$

$x_1 + 17x_2 + 4x_3 = 0,$

3.e. If u and v are any two vectors in an inner product space V. show that II u +v II^2 + II u - v II^2=2II u II^2 +2II v II^2. 6

3.f. Prove that 6

$U = \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is unitary matrix. CO 4

3.g.	Find the singular values of the matrix $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$. (CO5)	6

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a.	If $A = \begin{matrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{matrix}$ and $\text{Adj}(\text{Adj}A) = A$, find a. (CO1)	10
4-b.	If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (i) find A^{-1} (ii) Show that $A^3 = A^{-1}$. (CO1)	10

5. Answer any <u>one</u> of the following:-

5-a.	Define linear dependent and linear independent vectors.Check the given set of vectors (1,1,1),(1,2,3),(1,-2,5) and (2,-1,1) are linearly dependent or independent.Express the vector (1,-2,5) as a linear combination of other vectors.	10
5-b.	Test for consistency and solve the system: (CO2) $2x_1 + x_2 + 2x_3 + x_4 = 6$ $6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$ $4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$ $2x_1 + 2x_2 - x_3 + x_4 = 10$	10

6. Answer any <u>one</u> of the following:-

6-a.	Apply Gram- Schmidt process to transform the basis {(1,1,1), (0,1,1), (0,0,1)} into an orthonormal basis.(CO3)	10
6-b.	Apply Gram-Schmidt process to transform the standard basis S={1,x,x^2} into an orthonormal basis over [-1,1].(CO3)	10

7. Answer any <u>one</u> of the following:-

7-a.	If $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$. Show that A is a Hermitian matrix and iA is skew Hermitian matrix. (CO4)	10
7-b.	Show that the mapping $T: V_3(R) \to V_2(R)$ defined as $T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3, a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ into $V_2(R)$. (CO4)	10

8. Answer any <u>one</u> of the following:-

8-a. Find the singular values of the $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and find the SVD decomposition of A. (CO5) 10

8-b. Find a singular value decomposition of the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$. (CO5) 10