Printed Page:- 05 | Subject Code:- ACSE0306 / ACSEH0306

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: III - CARRY OVER THEORY EXAMINATION - AUGUST 2023

Subject: Discrete Structures

Time: 3 Hours | **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. The set O of odd positive integers less than 10 can be expressed by ____________ (CO1) 1

(a) {1, 2, 3}

(b) {1, 3, 5, 7, 9}

(c) {1, 2, 5, 9}

(d) {1, 5, 7, 9, 11}

1-b. A function is said to be ____________ if and only if f(a) = f(b) implies that a = b for all a and b in the domain of f. (CO1) 1

(a) One-to-many

(b) One-to-one

(c) Many-to-many

(d) Many-to-one

1-c. A function f:(M,□)→(N,×) is a homomorphism if ______ (CO2) 1

(a) f(a, b) = a*b

(b) f(a,b) = a / b

(c) f(a, b) = f(a)+f(b)

(d) f(a, b) = f(a)*f(a)

1-d. A cyclic group is always an (CO2) 1

(a) Ring

(b) Field

(c) Abelian Group

(d) Zero Ring

1-e. A free semilattice has the _______ property. (CO3) 1

(a) intersection

(b) commutative and associative

(c) universal

(d) identity

1-f. A POSET in which every pair of elements has both least upper bound and greatest lower bound term as. (CO3) 1

(a) Lattice

(b) Sublattice

(c) Walk

(d) POSET

1-g. $p \leftrightarrow q$ is logically equivalent to ________ (CO4) 1

(a) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

(b) $(p \rightarrow q) \vee (q \rightarrow p)$

(c) $(p \rightarrow q) \wedge (q \rightarrow p)$

(d) $(p \wedge q) \rightarrow (q \wedge p)$

1-h. What rules of inference are used in this argument? "It is either colder than Himalaya today or the pollution is harmful. It is hotter than Himalaya today. Therefore, the pollution is harmful." (CO4) 1

(a) Conjunction

(b) Modus ponens

(c) Disjunctive syllogism

(d) Hypothetical syllogism

1-i. The number of circuits that can be created by adding an edge between any two vertices in a tree is (CO5) 1

.

(a) Two

(b) Exactly one

(c) At least two

(d) None

1-j. A linear graph consists of vertices arranged in a line. (CO5) 1

(a) TRUE

(b) FLASE

(c) either true or false

(d) cannot determined

2. Attempt all parts:-

2.a. Define Equal and Equivalent Set with example. (CO1) 2

2.b. In a group (G, *) , Prove that $(a * b)^{-1} = b^{-1} * a^{-1}$, for all a,b is the element in G. (CO2) 2

2.c. How we can say a lattice to be a partial lattice . Justify with example. (CO3) 2

2.d. Prove that if x is irrational, then 1/x is irrational. (CO4) 2

2.e. Define Regular graph and Complete Bipartite graph with example. (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. Differentiate contraposition and contradiction with example. (CO1) 6

3-b. Give an example of two uncountable sets A and B such that A − B is (CO1) 6
a) finite.
b) countably infinite.
c) uncountable.

3-c. Let G be a finite group and let S be a non-empty set. Suppose that G acts on S freely and transitively. Prove that G=S. That is, the number of elements in G and S are the same. (CO2) 6

3-d. Let R=(R,+) be the additive group of real numbers and let R×=(R∖{0},⋅) be the multiplicative group of real numbers. (a) Prove that the map exp:R→R× defined by exp(x)=ex is an injective group homomorphism. (b) Prove that the additive group R is isomorphic to the multiplicative group R+={x∈R ∣ x>0}. (CO2) 6

3.e. Define POSET with example. Explain types of Lattice with suitable example. (CO3) 6

3.f. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty. (CO4) 6

a) $(\forall x P(x)) \wedge A \equiv \forall x(P(x) \wedge A)$
b) $(\exists x P(x)) \wedge A \equiv \exists x(P(x) \wedge A)$.

3.g. Explain the following: (CO5) 6
i. Directed Graph
ii. Weighted Graph
iii. Null Graph

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. State and prove both De'morgans laws. (CO1) 10

4-b. What is closure properties of relations, explain with example. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. find all the generators of cyclic group G= {1,2,3,4} with respect to operation X5. (CO2) 10

5-b. Let G and G' be groups and let $f:G \rightarrow G'$ be a group homomorphism. 10
If H' is a normal subgroup of the group G', then show that $H=f^{-1}(H')$ is a normal subgroup of the group G. (CO2)

6. Answer any <u>one</u> of the following:-

6-a. Show that in a complemented , distributive lattice the following are equivalent : (CO3) 10
(i). $a \wedge b' = 0$,
(ii). $a' \vee b = 1$

6-b. Consider the subset {2,3} {4,6} and {3,6}, and ({1,2,3,4,5,6}, /) is the poset. (CO3) 10
i) Draw the Hasse Diagram.
ii) Find the Lower bound and Upper bound of each subset if I exists.
iii) Find GLB and LUB of each subset if it exists.

7. Answer any <u>one</u> of the following:-

7-a. Let Q(x, y) denote the statement “x is the capital of y.” What are these truth values? (CO4) 10

a) Q(Denver, ¬Colorado)

b) Q(¬Detroit, Michigan)

c) Q(¬Massachusetts, ¬ Boston)
d) Q(New York, New York)

7-b. Find the dual of each of these compound propositions. (CO4) 10
a) $p \wedge \neg q \wedge \neg r$,
b) $(p \wedge q \wedge r) \vee s$,

.

c) $(p \vee F) \wedge (q \vee T)$

8. Answer any <u>one</u> of the following:-

8-a. For maximal planar graph G, prove or disprove the following: (i) if the number of vertices is less than or equal to 11 then G has minimum degree less than or equal to 4. (ii) if the number of vertices is greater than or equal to 4 then G has minimum degree greater than or equal to 3. (iii) every 5-connected maximal planar graph has at least 12 vertices. (CO5) 10

8-b. Suppose the characters 'a', 'b' , 'c' , 'd' , 'e' , 'f' , 'g' are stored in a Binary Search - Tree (BST). Draw a BST that is as tall as possible and contains all these characters. Also draw a BST that is as short as possible and contains all characters. (CO5) 10