Printed Page:- Subject Code:- AAS0104

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech.

SEM: I - CARRY OVER THEORY EXAMINATION - SEPTEMBER 2022

Subject: Mathematical Foundations-I

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. The characteristic roots of a real symmetric matrix are all (CO1) 1

(a) real

(b) imaginary

(c) pure imaginary

(d) none of these

1-b. The value of λ for which the vectors (1, -2, λ), (2, -1, 5) and (3, -5, 7λ) are linearly dependent. (CO1) 1

(a) 1

(b) 5/14

(c) 0

(d) None of these

1-c. Which condition exist for function to be linear transformation ? (CO2) 1

(a) $T(a\alpha + b\beta) = aT(\beta) + bT(\alpha)$

(b) $T(a\alpha + b\beta) = aT(\alpha) + bT(\alpha)$

(c) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$

(d) None of these

1-d. If T be a linear transformation from U into V , then according to rank and nullity theorem: (CO2) 1

(a) Rank(T) - Nullity(T) = Dim U

(b) Rank(T) + Nullity(T) = Dim U

(c) Rank(T) + Nullity(T) = Dim V

(d) None of these

1-e. If even power of y then curve is symmetrical about the line (CO3) 1

(a) y- axis

(b) x- axis

(c) x and y both axis

(d) line y=x

1-f. If $z = xy\, f\left(\frac{x}{y}\right)$ then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial x}$ is (CO3) 1

(a) 0

(b) z

(c) 2z

(d) 3z

1-g. With usual notation a function f(x, y) has a saddle point at (a, b) if (CO4) 1

(a) $rt - s^2 < 0$

(b) $rt - s^2 > 0$

(c) $rt - s^2 = 0$

(d) $rt = s$

1-h. If $u = x + y + z,\ v = 2x + 2y + 2z,\ w = \frac{1}{2}(x + y + z)$ are functionally dependent then the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ is (CO4) 1

(a) 1

(b) 2

(c) 3

(d) 0

1-i. A man had 7 children. When their average age was 12 years, a child aged 6 years died. The average age of remaining six children is (CO5) 1

(a) 13 years

(b) 10 years

(c) 11 years

(d) 14 years

1-j. If blue is coded as green, green is coded as white and white is code as black, and then what will be the code for the colour of grass? (CO5) 1

(a) White

(b) Green

(c) Black

(d) None of These

2. Attempt all parts:-

2.a. Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into normal form and find its rank. (CO1) 2

2.b. The subset S { (1, 0, 0), (0, 1, 0), (0, 0, 1) } of the vector space R^3 is linearly..... (CO2) 2

2.c. Find the n^{th} differential coefficients of $x^2 e^x$ (CO3) 2

2.d. What is the maximum value of the function $f(x,y) = 1-x^2-y^2$. (CO4) 2

2.e. If "PROMPT" is coded as QSPLOS, then "PLAYER" should be? (CO5) 2

SECTION B 30

3. Answer any <u>five</u> of the following:-

3-a. Find the rank of matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 1 & 2 & -8 \end{bmatrix}$ (CO1) 6

3-b. If $\begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$. Show that A is hermitian matrix and iA is skew-Hermitian matrix. (CO1) 6

3-c. Determine whether or not the following vectors form a basis of R^3 : (1, 1, 2), (1, 2, 5), (5, 3, 4). (CO2) 6

3-d. If α and β are vectors in an inner product space then show that $\|\alpha+\beta\|^2 + \|\alpha-\beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$. (CO2) 6

3.e. Find the asymptotes of the curve $4x^3 - x^2y - 4xy^2 + y^3 + 3x^2 + 2xy - y^2 - 7 = 0$. (CO3) 6

3.f. Examine the function $f(x,y) = x^3 + y^3 - 3axy$ for maximum and minimum values. (CO4) 6

3.g. In an examination, 34% of the students failed in Mathematics and 42% failed in English. If 6

20% of students failed in both the subjects, then the percentage of students who passed was. (CO5)

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (CO1) 10

4-b. Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence compute A^{-1}. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Show that the mapping $T: R^2 \to R^3$ defined as $T(a, b) = (a - b, b - a, -a)$ is a linear transformation. Find the range, null-space and nullity of T. (CO2) 10

5-b. Let $v = R^2 = \{(x, y): x, y \in R\}$ *and* F = R. Define the addition and scalar multiplication in R^2 as follows $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ *and* $a(x, y) = (ax, ay)$. Show that R^2 is a vector space over R.(CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. Trace the curve $y^2(2a - x) = x^3$ (CO3) 10

6-b. If $y = e^{a \sin^{-1} x}$, then prove that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + a^2) y_n = 0$ (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Use the method of Lagrange's multiplier to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{z^2} = 1$. (CO4) 10

7-b. In estimating the number of bricks in a pile which is measured to be (5 m x 10 m x 5 m) the count of bricks is taken as 100 bricks/meter^3. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is Rs. 2000 per thousand bricks. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) If in certain code 1326 is coded as 8673, and 5670 is coded as 4329, then the code for 0009 will be 10

(ii) The total population of a village is 5000. The number of male and female increases by 10% and 15% respectively and consequently the population of the village become 5600. What was the number of males in the village?

(iii) A dealer offers a discount of 10% on the marked price of an article and still makes a profit of 20%. If its marked price is Rs. 800, then the cost price is? (CO5)

8-b. (i) If the price of an item is decreased by 10% and then increased by 10%, the net effect on the price of the item is:(CO5) 10

(ii) The average marks obtained by 40 students of a class is 86. If the 5 highest marks are removed, the average reduced by one marks. The average marks of the top 5 students is ?(CO5)

(iii) Find the missing terms: 1, 2, 6, 7, 21, 22, 66, 67, ? (CO5)