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Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - CARRY OVER THEORY EXAMINATION - SEPTEMBER 2022

Subject: Linear Algebra

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 marks each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. **Find the adjoint of a matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.** (CO1) 1

(a) $\begin{bmatrix} 5 & 10 \\ 3 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -2 \\ -3 & -1 \end{bmatrix}$

(d) None of these

1-b. **The matrix $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is** (CO1) 1

(a) Nilpotent matrix

(b) Idempotent matrix

(c) Involuntary matrix

(d) None of these

1-c. Find the value of μ such that the rank of the matrix $\begin{bmatrix} \mu & -1 & 0 \\ 0 & \mu & -1 \\ -1 & 0 & \mu \end{bmatrix}$ is 2. (CO2) 1

(a) 1

(b) 2

(c) 3

(d) none of these

1-d. The characteristic roots of a real symmetric matrix are all (CO2) 1

(a) real

(b) imaginary

(c) pure imaginary

(d) none of these

1-e. Which of the set of vectors are linearly dependent? (CO3) 1

(a) (1, 1, -1), 2, -3, 5), (-2, 1, 4)

(b) (1, -1, -1), (2, -3, 5), (-2, 1, 4)

(c) (1, 4, -1), 2, -2, 5), (-2, 1, 4)

(d) None of these

1-f. A subset W is called subspace of vector space V(F) for a, b ∈ F and α, β ∈ V, is satisfy -(CO3) 1

(a) $a\alpha - b\beta \in V$

(b) $a\alpha \times b\beta \in V$

(c) $a\alpha \div b\beta \in V$

(d) $a\alpha + b\beta \in V$

1-g. A square matrix A is positive definite if it is symmetric and (CO4) 1

(a) $x^T Ax > 0$

(b) $x^T Ax = 0$

(c) $x^T Ax < 0$

(d) None of these

1-h. The eigen values of $4A^{-1} + 3A + 2I$, where $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ are (CO4) 1

(a) 1, 2

(b) 9, 15

(c) 3, 4

(d) None of these

1-i. If $A=\begin{bmatrix}3 & 2\\1 & 2\end{bmatrix}$ then Eigen values of A^3 .(CO5) 1

(a) 1,4

(b) 1,64

(c) 4,4

(d) None of these

1-j. The sum of Eigen values of a square matrix is equal (CO5) 1

(a) The sum of the elements of its principal diagonal

(b) The product of the elements of its principal diagonal

(c) The sum of all elements of the matrix

(d) None of these

2. Attempt all parts:-

2.a. If $A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, show that $A^2-5A=2I$, where I is the unit matrix of order 2.(CO1) 2

2.b. Find the value of λ for which the vectors $X_1=(1,\ -2,\ \lambda), X_2=(2,\ -1,\ 5), X_3=(3,\ -5,\ 7,\ \lambda)$ are linearly dependent.(CO2) 2

2.c. Show tha the three vectors $\{(1, 1, \ -1), (2, \ -3, 5), (-2, 1, 4)\}$in R^3 are linearly independent.(CO3) 2

2.d. Obtain the eigen value of $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$. (CO4) . 2

2.e. Find the singular values of matrix $A=\begin{bmatrix}1 & 3\\3 & 1\end{bmatrix}$.(CO5) 2

SECTION B 30

3. Answer any five of the following:-

3-a. Find the inverse of the matrix $\begin{bmatrix}2 & 3 & 4\\4 & 3 & 1\\1 & 2 & 4\end{bmatrix}$ by elementary transformation.(CO1) 6

3-b. Solve the following equations by Cramer's rule- (CO1) 6

x+y+z=6, 2x+3y-z=5 and 6x-2y-3z=-7

3-c. Find the LU decomposition of a matrix $\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}$.(CO2) 6

3-d. Find the values of a and b such that the rank of matrix $\begin{bmatrix}1 & -2 & 3 & 1\\2 & 1 & -1 & 2\\6 & -2 & a & b\end{bmatrix}$ is 2.(CO2) 6

3.e. Show that the vectors (2,1,4), (1,-1,2) and (3,1,-2) forms a basis of R^3.(CO3) 6

3.f. Find the eigenvalues and eigenvectors of a matrix of $A=\begin{bmatrix}1 & 0\\ 1 & 2\end{bmatrix}$. (CO4) 6

3.g. Find a singular value decomposition of the matrix $A=\begin{bmatrix}2 & -1\\ 2 & 2\end{bmatrix}$.(CO5) 6

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. Solve the system of equations by matrix method: (CO1) 10

x+2y-3z=4, 2x+3y+2z=2 and 3x-3y-4z=11.

4-b. Solve the following equations by matrix inversion method.(CO1) 10

$x+y+z=4,\ x-y+z=0,\ 2x+y+z=5.$

5. Answer any <u>one</u> of the following:-

5-a. Find the rank of a matrix reducing to normal form $A=\begin{bmatrix}2 & 1 & -3 & -6\\ 3 & -3 & 1 & 2\\ 1 & 1 & 1 & 2\end{bmatrix}$.(CO2) 10

5-b. Solve the following system of equation by LU decomposition method:(CO2) 10

$x_1+x_2+x_3=1,\ 4x_1+3x_2-x_3=6,\ 3x_1+5x_2+3x_3=4.$

6. Answer any <u>one</u> of the following:-

6-a. Show that the transformation T: $V_2(R)\rightarrow V_3(R)$ defined as 10

T(a, b)=(a+b, a-b, b) $\forall\ a,b \in R$ is linear. Find its null space, nullity, range and rank.(CO3)

6-b. Apply Gram-schmidt process to the vectors α_1=(1,0,1), α_2=(1,0,-1), α_3=(0,3,4) to obtain the orthonormal basis for $V_3(R)$.(CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Find the eigen values and eigen vector of the matrix $A=\begin{bmatrix}3 & 0 & 0\\ 1 & -2 & -8\\ 0 & -5 & 1\end{bmatrix}$. (CO4) 10

7-b. Is the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1+i\\ 1-i & -1\end{bmatrix}$ is unitary? (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. Find a singular value decomposition of $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$.(CO5) 10

8-b. Given the following data, use PCA to reduce the dimension from 2 to 1.(CO5) 10

Feature	Example 1	Example 2	Example 3	Example 4
x:	4	8	13	7
y:	11	4	5	14