NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech. (Integrated)

SEM: I - CARRY OVER THEORY EXAMINATION - SEPTEMBER 2022

Subject: Engineering Mathematics-I

Time: 3 Hours   Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.

2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.

3. Section B - Question No-3 is based on external choice carrying 6 marks each.

4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.

5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A   20

1. Attempt all parts:-

1-a.   The rank of matrix $\begin{bmatrix} 5 & 10 \\ 3 & 6 \end{bmatrix}$ is (CO1)   1

        (a) 2

        (b) 3

        (c) 0

        (d) 1

1-b.   If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are (CO1)   1

        (a) $4, 5^2, 7^3$

        (b) 4, 5, 7

        (c) 1/4, 1/5, 1/7

        (d) none of these

1-c.   The nth derivative of cos(ax+b) is (CO2)   1

        (a) $a^n \cos(ax+b)$

        (b) $a^n \cos\left(ax+b+\dfrac{n\pi}{2}\right)$

        (c) $a^n \cos\left(ax+b+\dfrac{n\pi}{4}\right)$

(d) None of these

1-d.   The asymptotes parallel to the y-axis of the curve $x^2 y^2 - a^2 (x^2 + y^2) = 0$ is (CO2)   1

        (a) 2a

        (b) $x = a$

        (c) $x = 3a$

        (d) None of these

1-e.   Maclaurin's series for f(x) is   (CO3)   1

        (a) $f(0) + \dfrac{x}{1!}f'(0) + \dfrac{x^2}{2!}f''(0) + \dfrac{x^3}{3!}f'''(0) + \ldots\ldots$

        (b) $f(x) + \dfrac{x}{1!}f'(x) + \dfrac{x^2}{2!}f''(x) + \dfrac{x^3}{3!}f'''(x) + \ldots\ldots$

        (c) $f(0) + \dfrac{x}{1}f'(0) + \dfrac{x^2}{2}f''(0) + \dfrac{x^3}{3}f'''(0) + \ldots\ldots$

        (d) $f(x) + \dfrac{1}{1!}f'(x) + \dfrac{1}{2!}f''(x) + \dfrac{1}{3!}f'''(x) + \ldots\ldots$

1-f.   If $u = x^2$ and $v = y^2$ then value of $\dfrac{\partial(u,v)}{\partial(x,y)}$ is   (CO3)   1

        (a) 4xy

        (b) 6xy

        (c) 2xy

        (d) xy

1-g.   The value of $\Gamma n\, \Gamma(1-n)$ is   (CO4)   1

        (a) $\dfrac{\pi}{\sin n\pi}$

        (b) $\dfrac{2\pi}{\sin n\pi}$

        (c) $\dfrac{\pi}{\cos n\pi}$

        (d) None of these

1-h.   The volume of the region bounded by $-1 \le x \le 1,\ -2 \le y \le 2,\ -3 \le z \le 3$ is   (CO4)   1

        (a) 24

        (b) 48

        (c) -24

        (d) -48

1-i. If two successive discount are 30% and 10%. Find single equivalent discount. (CO5) 1

      (a) 35%

      (b) 39%

      (c) 41%

      (d) None

1-j. If blue is coded as green, green is coded as white and white is code as black, and then what will be the code for the colour of grass? (CO5) 1

      (a) White

      (b) Green

      (c) Black

      (d) None of These

2. Attempt all parts:-

2.a. Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into normal form and find its rank. (CO1) 2

2.b. Find the nth differential coefficients of $x^2\, e^x$. (CO2) 2

2.c. If the base radius and height of a cone are measured as 4 and 8 inches with a possible error of 0.04 and 0.08 inches respectively, calculate percentage error in calculating volume of the cone. (CO3) 2

2.d. Evaluate the value of $\int_0^1 x^4(1-x)^3\,dx$. (CO4) 2

2.e. The average age of 10 children is 9 years 9 months. The average age of 9 children is 8 years 11 months. What is the age of the tenth child ? (CO5) 2

<div align="center">SECTION B   30</div>

3. Answer any <u>five</u> of the following:-

3-a. Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary transformation. (CO1) 6

3-b. Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and verify Caley-Hamilton theorem for this matrix. Find A $^{-1}$ also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. (CO1) 6

3-c. Trace the following curve $y^2 (a - x) = x^2 (a + x)$. (CO2) 6

3-d. If $u = \sin^{-1}\left(\dfrac{x + 2y + 3z}{\sqrt{x^8 + y^8 + z^8}}\right)$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial}{\partial} = -3 \tan u$. (CO2) 6

3.e. Expand $f(x,y) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's series expansion about the point (1,2) upto three terms. (CO3)   6

3.f. Evaluate by changing the order of integration $\int_0^\infty \int_0^x x e^{-x^2/y} \, dy \, dx$. (CO4)   6

3.g. A machine is sold for Rs 5060 at a gain of 10%. What would have been the gain or loss % if it had been sold for Rs 4370? (CO5)   6

<center>SECTION C   50</center>

4. Answer any <u>one</u> of the following:-

4-a. Determine the value of b such that system of homogeneous equation $\begin{aligned} 2x + y + 2z &= 0 \\ x + y + 3z &= 0 \\ 4x + 3y + bz &= 0 \end{aligned}$ has   10

(i) trivial solution (ii) non-trivial solutions. Find the non-trivial solutions using matrix method.(CO1)

4-b. Find the eigen values and eigen vectors of a matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (CO1)   10

5. Answer any <u>one</u> of the following:-

5-a. If $y = a\cos(\log x) + b\sin(\log x)$, then show that
$x^2 y_2 + x y_1 + y = 0$ and $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (CO2)   10

5-b. If $y = \sin(a \sin^{-1} x)$, then find $y_n(0)$. (CO2)   10

6. Answer any <u>one</u> of the following:-

6-a. If $u = \dfrac{x+y}{z}$, $v = \dfrac{y+z}{x}$, $w = \dfrac{y(x+y+z)}{xz}$, then show that u, v, w are not independent and find the relation between them. (CO3)   10

6-b. A rectangular box closed at the top is of given volume, what must be the dimensions so that the surface area is minimum. (CO3)   10

7. Answer any <u>one</u> of the following:-

7-a. Prove that $\Gamma(m)\,\Gamma(m+1/2) = \dfrac{\sqrt{\pi}}{2^{2m-1}\,\Gamma(2m)}$, where m is a positive number. (CO4)   10

7-b. Apply Dirichlet theorem to evaluate mass of the tetrahedron OABC for the plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ that meets the coordinate axis at A,B and C, if the density at a point is kxyz. (CO4)   10

8. Answer any <u>one</u> of the following:-

8-a. (i) A sells a suitcase to B at 10% profit. B sells it to C at 30% profit. If C pays Rs 2860 for it, then find the cost price for A ?   10

(ii) If PARK is coded as 5394, SHIRT is coded as 17698 and PANDIT is coded as 53068, how would you code NISHAR in that code language?

(iii) Out of four numbers the average of the first three is 16 and that of the last three is 15. If the last number is 20 then find the first number?. (CO5)

8-b.   (i) The average age of husband, wife and their child 3 years ago was 27 years and that of   10
wife and the child 5 years ago was 20 years. Find the present age of husband?

(ii) A tradesman sold an article at a loss of 20%. If the selling price had been increase by Rs. 100, there would have been a gain of 5%. Find the cost price of the article?

(iii) Find the missing term
      122, 62, 32, ?, 9.5, 5.75 . (CO5)