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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech (Integrated)

SEM: II - CARRY OVER THEORY EXAMINATION - SEPTEMBER 2022

Subject: Engineering Mathematics-II

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 marks each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. Degree and order of the differential equation $x\frac{d^2y}{dx^2}+y\frac{dy}{dx}+4y^2=1$ (CO1) 1

(a) Ord = 2, Deg = 2

(b) Ord = 1, Deg = 2

(c) Ord = 2, Deg = 1

(d) Ord = 2, Deg = 0

1-b. General solution of the second order linear differential equation $(d^2y/dx^2)-8(dy/dx)+16y=0$ (CO1) 1

(a) $(A+Bx)e^{4x}$

(b) $A+Be^{4x}$

(c) $Ae^{-4x}+Be^{4x}$

(d) $A-Bx^{4x}$

1-c. The coefficient 'a_0' in a Fourier series for the function f(x) = x in the interval 0 < x < 2 π is (CO2) 1

(a) π

(b) 0

(c) $2\pi^2$

(d) 2π

1-d. For the series $\sum_{n=1}^{\infty} u_n$ of positive terms the Ratio test fails if $\lim_{n\to\infty} \frac{u_n}{u_{n+1}}$ is (CO2) 1

(a) <1

(b) >1

(c) = 1

(d) none of these

1-e. Laplace transform of $f(t) = 5e^{2t} + 2$ is (CO3) 1

(a) $\frac{5}{s-2} - \frac{1}{s}$

(b) $\frac{5}{s-2} + \frac{1}{2s}$

(c) $\frac{5}{s-2} + \frac{2}{s}$

(d) None of these

1-f. Inverse Laplace of the function $f(s) = \frac{e^{-2s}}{s}$ (CO3) 1

(a) $u(t-2)$

(b) $u(t+2)$

(c) $-u(t+2)$

(d) $-u(t-2)$

1-g. If $\vec{F} = (y+z)\,\hat{i} + (z+x)\,\hat{j} + (x+y)\,\hat{k}$, curl of $\vec{F}$ equal to (CO4) 1

(a) 0

(b) $\hat{i}+\hat{j}+\hat{k}$

(c) 3

(d) None of these

1-h. Find the unit normal at the surface $z = x^2 + y^2$ at the point $(1,2,5)$ is (CO4) 1

(a) $2x + 4y$

(b) $2x\,\hat{i} + 4y\,\hat{j}$

(c) $\frac{-2x\,\hat{i} - 4y\,\hat{j} + \hat{k}}{\sqrt{21}}$

(d) None of these

1-i. A boy has coins in the denominations of ₹ 1 and ₹ 2. If he has total 30 coins and 1

the value of coins is ₹ 48. Find the number of ₹ 1 coins he has. (CO5)

(a) 18

(b) 10

(c) 12

(d) 14

1-j. The simple interest on a certain sum of money at 4% per annum for 4 years is Rs 80 more than the simple interest on the same sum of money for 3 years at 5% per annum. Find the sum ? (CO5) 1

(a) Rs 4000

(b) Rs 8000

(c) Rs 4030

(d) none of these

2. Attempt all parts:-

2.a. Find the Particular integral of the differential equation $(4D^2 + 4D - 3)y = e^{2x}$ (CO1) 2

2.b. Discuss the convergence of the sequence $\{a_n\}$ where $a_n = \left(\frac{n+1}{n}\right)$.(CO2) 2

2.c. Find Laplace transform of the function $F(t) = te^{-4t}\sin 3t$.(CO3) 2

2.d. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$,then show that grad $(r) = \frac{\vec{r}}{r}$. (CO4) 2

2.e. After 2 years, the age of Karthi is 2 times the present age of Silambu. Preethi is 8 years elder than Silambu. Find the present age of Karthi, if the present age of Preethi is 23 years? (CO5) 2

SECTION B 30

3. Answer any <u>five</u> of the following:-

3-a. Solve $\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t$, $\frac{dx}{dt} + y = \cos t$.(CO1) 6

3-b. Solve the following differential equation: 6

$x^2y'' + xy' - y = x^2e^x$. (CO1)

3-c. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \ldots\ldots\ldots$(CO2) 6

3-d. Expand f(x) = π x - x^2 as a Fourier half range sine series in 0 < x < π upto the first three terms. (CO2) 6

3.e. Evaluate the value of the integral $\int_0^\infty \frac{e^{-t}\sin t}{t}dt$.(CO3) 6

3.f. Find the directional derivative of the function $f = (x^2+y^2+x^2)^{-1/2}$ at $(3,1,2)$ in direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$. (CO4) 6

3.g. (i) A girl leaves from her home. She first walks 30 m in North-West direction and then 30 m 6

in South-West direction. Next, she walks 30 m in South-East direction. Finally, she turns towards her house. In which direction is she moving?

(ii) Kashish goes 30m North, then turns right and walks 40 m, then again turns right and walks 20 m, then again turns right and walks 40 m. How far is he from his original position? (CO5)

SECTION C 50

4. Answer any one of the following:-

4-a. Solve the differential equation in series : $2x(1-x)y'' + (1-x)y' + 3y = 0$ (CO1) 10

4-b. Solve the differential equations by method of variation of parameters $y'' - y = \frac{2}{1+e^x}$. (CO1) 10

5. Answer any one of the following:-

5-a. Test the convergence of the series, $x + \frac{1.x^3}{2.3} + \frac{1.3.x^5}{2.4.5} + \frac{1.3.5\,x^3}{2.4.6\,7} + \ldots\ldots\ldots$ (CO2) 10

5-b. Obtain the Fourier series to represent function $f(x) = x + x^2$ in the interval $-\pi \le x \le \pi$. (CO2) 10

Hence show that $(i)\ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{6}$

$(ii)\ \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$

6. Answer any one of the following:-

6-a. Solve the following differential equation by using Laplace transform $\frac{d^2y}{dt^2} + y = t\cos 2t$, *Given that* $y(0) = y'(0) = 0$. (CO3) 10

6-b. By using Convolution theorem, find $\mathbf{L}^{-1}\left\{\frac{s}{(s^2+4)(s^2+9)}\right\}$. (CO3) 10

7. Answer any one of the following:-

7-a. Apply Stokes theorem to evalaute $\int_C (x+y)\,dx + (2x-z)\,dy + (y+z)\,dz$, where C is the boundary of the triangle with vertices (2,0,0), (0,3,0), (0,0,6). (CO4) 10

7-b. Evaluate by using the Divergence theorem $\iint_S \vec{F}.\hat{n}\,dS$, where $\vec{F} = (x+y^2)\hat{\imath} - 2x\hat{\jmath} + 2yz\hat{k}$ and S is the surface of the plane 2x+y+2z=6 in the first octant. (CO4) 10

8. Answer any one of the following:-

8-a. (i) Compound interest on a sum of money for 2 years at 4 per cent per annum is Rs.2448. Find the simple interest of the same sum of money at the same rate of interest for 2 years? 10

(ii) The simple interest on a sum of money at 4% per annum for 2 years is 80. Find the compound interest in the same sum for the same period? (CO5)

8-b. (i) Three vessels containing mixtures of milk and water are of capacities which are in the ratio 1:2:3. The ratios of milk and water in the three vessels are 4:1, 3:2 and 2:3 respectively. If one-fourth contents of first vessel, one-third of that of second vessel and half of that of third vessel are mixed; what is the ratio of milk and water in the new mixture? (CO5) 10

(ii) A, B and C enter into a partnership by investing 1500, 2500 and 3000 rupees respectively. A as manager gets one-tenth of the total profit and remaining profit is divided among the three in the ratio of their investment. If A's total share is Rs. 369, find the shares of B and C. (CO5)