Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2021 - 2022)

Subject: Engineering Mathematics-II

Time: 3 Hours   Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.

2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 marks each.

3. Section B - Question No-3 is based on external choice carrying 6 marks each.

4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.

5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

<div align="center">SECTION A   20</div>

1. Attempt all parts:-

1-a.   Degree and order of the differential equation $\sqrt{(dy/dx)^2 + 3y} = (d^2y)/(dx^2)$ is   (CO1)   1

                 (a) Ord = 2, Deg = 2

                 (b) Ord = 2, Deg = 1

                 (c) Ord = 1, Deg = 2

                 (d) Ord = 1, Deg = 1

1-b.   The P.I. of the differential equation $(D^2 + 6D + 9)y = 5e^{3x}$ (CO1)   1

                 (a) $(5/36)\,e^{3x}$

                 (b) $(1/36)\,e^{3x}$

                 (c) $(5x/36)\,e^{3x}$

                 (d) None of these

1-c.   The Series $1 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{4}} + \dfrac{1}{\sqrt{5}} + \ldots\ldots$ is   1

                                            (CO2)

                 (a) convergent

                 (b) oscillatory

                 (c) divergent

                 (d) none of these

1-d.   The coefficient a_0 in a Fourier series for the function $f(x) = x + x^3$ in the interval $-\pi < x <$ 1

π is   (CO2)

    (a) π

    (b) 2π

    (c) 0

    (d) none of these

1-e.   Laplace transform of te^{-t} is   (CO3)   1

    (a) $\dfrac{1}{(s+1)^2}$

    (b) $\dfrac{2}{(s+1)^2}$

    (c) $\dfrac{1}{(s-1)^2}$

    (d) None of these

1-f.   Laplace transform of $e^{-3t}u(t-2)$ is   (CO3)   1

    (a) $\dfrac{e^{-2(s+3)}}{s-3}$

    (b) $\dfrac{e^{-2(s+3)}}{s+3}$

    (c) $\dfrac{e^{-2(s+3)}}{s^2+3}$

    (d) $\dfrac{e^{-2(s+3)}}{s^2-3}$

1-g.   The directional derivative of $\dfrac{1}{r}$ in the direction of $\vec{r}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is   (CO4)   1

    (a) $-\dfrac{1}{r^2}$

    (b) $-\dfrac{5}{r^4}\vec{r}$

    (c) $-\dfrac{2}{r^5}\vec{r}$

    (d) $\dfrac{8}{r^4}\vec{r}$

1-h.   Divergence of $2x^2z\hat{i} - xy^2z\,\hat{j} + 3yz^2\hat{k}$ at the point $(1,1,1)$ is   (CO4)   1

    (a) 0

    (b) 12

    (c) 3

    (d) 8

1-i. A person starts towards South direction. Which of the following order of direction will lead him to East direction? (CO5) 1

    (a) Right, Right, Right

    (b) Left, Left, Left

    (c) Left, Right, Right

    (d) Right, Left, Right

1-j. Find the simple interest on Rs 500 for 5 years at 10% per annum. (CO5) 1

    (a) Rs 500

    (b) Rs 125

    (c) Rs 250

    (d) Rs 350

2. Attempt all parts:-

2.a. Solve the differential equation: $(D^2 - 2D + 4)^2 y = 0$. (CO1) 2

2.b. Test the convergence of the series $\sum_{n=1}^{\infty} u_n$ where $u_n = \dfrac{1 + (n+1)}{1 + (n+1)^2}$. (CO2) 2

2.c. Find the inverse Laplace transform of the function $f(s) = \dfrac{1}{s^{3/2}} + \dfrac{1}{s-4}$. (CO3) 2

2.d. Show that vector $\vec{V} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$, is solenoidal. (CO4) 2

2.e. If P \$ Q means P is the father of Q; P # Q means P is the mother of Q and P * Q means P is the sister of Q, then N # L \$ P * Q shows which of the relation of Q to N? (CO5) 2

<div align="center">SECTION B   30</div>

3. Answer any <u>five</u> of the following:-

3-a. Solve the differential equation $x^2 \dfrac{d^2 y}{dx^2} + 4x \dfrac{dy}{dx} + 2y = e^x$ (CO1) 6

3-b. Solve $\dfrac{dx}{dt} - y = e^t, \dfrac{dy}{dt} - x = \sin t; x(0) = 1, y(0) = 0.$ (CO1) 6

3-c. Test the convergence of the series $\sum_{n=1}^{\infty} \dfrac{(n+1)}{n^3} x^n$ (CO2) 6

3-d. Expand $f(x) = \pi x - x^2$ as a Fourier half range sine series in $0 < x < \pi$ upto the first three terms. (CO2) 6

3.e. Find the Laplace Transform of the function $F(t) = \int_0^t te^{-t} \sin 4t \, dt$. (CO3) 6

    6

3.f   Find the total work done by a force $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ in moving a point from (0,0) to (a,b) along the rectangle bounded by the lines x=0, x=a, y=0, and y=b.   (CO4)

3.g.   (i) Lalit along with his family decided to take a road trip to a nearby resort and spend the   6
weekend there. He started from his home and from there drove 70 km to the south, he then took a right turn and drove 30 km. Next, he took a right turn and drove 30 km and stopped at a restaurant. What is the shortest distance between his house and the restaurant?   (CO5)
(ii) Two customers borrowed the same amount of money, one at compound interest and at the other at simple interest. If after 2 years, the interest payable by one was Rs. 220 and the other Rs. 200, then what was the principle money lent to each of them?   (CO5)

<div align="center">SECTION C   50</div>

4. Answer any <u>one</u> of the following:-

4-a.   Solve the differential equation in series $xy'' + y' - y = 0$ about $x = 0$.   (CO1)   10

4-b.   Solve the differential equations by method of variation of parameters
Solve the differential equations by method of variation of parameters $y'' - y = \dfrac{2}{1+e^x}$.   (CO1)   10

5. Answer any <u>one</u> of the following:-

5-a.   Test   the   convergence   the   series,   10

$$1 + \frac{2}{3} \cdot \frac{1}{4} + \frac{2.4}{3.5} \cdot \frac{1}{6} + \frac{2.4.6}{3.5.7} \cdot \frac{1}{8} + \frac{2.4.6.8}{3.5.7.9} \cdot \frac{1}{10} + \dots \dots \quad (CO2)$$

5-b.   Obtain the Fourier series to represent function f(x) = x sinx in the interval $-\pi \le x \le \pi$.   10
Hence show that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots \dots = \frac{\pi - 2}{4}.$$ (CO2)

6. Answer any <u>one</u> of the following:-

6-a.   Solve   the   following   differential   equation   by   using   Laplace   transform   10
$y'' + 3y' + 2y = te^{-t}$, $where \; y(0) = 1, y'(0) = 0$.   (CO3)

6-b.   By using Convolution theorem, find $L^{-1}\left\{ \dfrac{s}{(s^2 + 4)(s^2 + 9)} \right\}$.   (CO3)   10

7. Answer any <u>one</u> of the following:-

10

7-a.   Apply Stokes theorem to evalaute $\displaystyle\int_C (x+y)\, dx + (2x - z)\, dy + (y + z)\, dz,$ where C
is the boundary of the triangle with vertices (2,0,0), (0,3,0), (0,0,6).   (CO4)

7-b. Verify Divergence theorem for $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ taken over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.   (CO4)   10

8. Answer any <u>one</u> of the following:-

8-a. (i) The present ages of three persons, Raj, Rajesh and Ravi is in proportions 4: 7: 9. Eight   10
years ago, the sum of their ages was 56. What is the present age of Ravi?   (CO5)

(ii) The ratio between two numbers is 3:4. If each number is increased by 2, the ratio
becomes 7:9. Find the numbers.   (CO5)

8-b. (i) Three vessels containing mixtures of milk and water are of capacities which are in the   10
ratio 1:2:3. The ratios of milk and water in the three vessels are 4:1, 3:2 and 2:3 respectively.
If one-fourth contents of first vessel, one-third of that of second vessel and half of that of
third vessel are mixed; what is the ratio of milk and water in the new mixture?   (CO5)

(ii) A, B and C enter into a partnership by investing 1500, 2500 and 3000 rupees
respectively. A as manager gets one-tenth of the total profit and remaining profit is divided
among the three in the ratio of their investment. If A's total share is Rs. 369, find the shares
of B and C.   (CO5)