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Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2021 - 2022)

Subject: Mathematical Foundations – II

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. What is the area of the region bounded by the curves x=0,y=0 and x+y=1 ? (CO1) 1

(a) 1/2

(b) 2

(c) 3/2

(d) π

1-b. The value of $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dx\, dy\, dz$ is (CO1) 1

(a) $(e-1)^2$

(b) $(e+1)^2$

(c) $(e-1)^3$

(d) None of these

1-c. General solution of the differential equation $(D^3-2D+4)y=0$ is (CO 2) 1

(a) $c_1e^{-2x}+x(c_2\cos x + c_3\sin x)$

(b) $c_1e^{-2x}+e^{x}(c_2\cos x + c_3\sin x)$

(c) $c_1e^{-2x}+e^{-x}(c_2\cos x + c_3\sin x)$

(d) $c_1 e^{-2x} + e^{2x}(c_2 \cos x + c_3 \sin x)$

1-d. Part of the C.F. of $xy'' - y' + (1-x)y = x^2 e^{-x}$ by the method of reduction of order is (CO 2) 1

(a) e^{-x}

(b) e^{x}

(c) x

(d) x^2

1-e. Which of the following is the correct partial differential equation of the relation z = (x + a)(y + b), where 'a' and 'b' are constant (CO3) 1

(a) $z = pq$

(b) $z = p + q + ab$

(c) $z = px + qy + pq$

(d) $z = px + qy - pq$

1-f. The linear partial differential equation $2\frac{\partial^2 \mathbf{u}}{\partial \mathbf{t}^2} + 4\frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}\partial \mathbf{t}} + 3\frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2} = 0$ is (CO3) 1

(a) Parabolic

(b) Elliptic

(c) Hyperbolic

(d) None of these

1-g. Inverse Laplace of the function $f(s) = \left[\frac{1}{s(s^2+1)}\right]$ is (CO 4) 1

(a) $1 - \cos t$

(b) $1 + \sin t$

(c) $1 - \sin t$

(d) None of these

1-h. Laplace transform of $e^{-3t}u(t-2)$ is (CO 4) 1

(a) $\frac{e^{-2(s+3)}}{s-3}$

(b) $\frac{e^{-2(s+3)}}{s+3}$

(c) $\frac{e^{-2(s+3)}}{s^2+3}$

(d) $\frac{e^{-2(s+3)}}{s^2-3}$

1-i. Introducing a boy, a girl said, “He is the son of the daughter of the father of my uncle.” How is the boy related to the girl? (CO5) 1

(a) Brother

(b) Nephew

(c) Uncle

(d) Son-in-law

1-j. If 2P = 3Q = 4R, Then P: Q: R =? (CO5) 1

(a) 2:3:5

(b) 2:3:4

(c) 6:4:3

(d) 6:2:3

2. Attempt all parts:-

2.a. Evaluate the integral $\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx$. (CO1) 2

2.b. Find the particular integral of differential equation $(D^2+4D+8)y = sin(2x+3)$. (CO2) 2

2.c. Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(x)$. (CO3) 2

2.d. Find Laplace transform of the function F(t) = cosh at cos at. (CO 4) 2

2.e. Sanjeev walks 10 m towards the South. Turning to the left, he walks 20 m and then moves to his right. After moving a distance of 20 m, he turns to the right and walks 20 m. Finally, he turns to the right and moves a distance of 10 m. How far and in which direction is he from the starting point? (CO5) 2

SECTION B 30

3. Answer any five of the following:-

3-a. Evaluate the integral by changing the order of integration $\int_0^a \int_{x^2/a}^{2a-x} \mathbf{xy\ dydx}$. (CO1) 6

3-b. Prove that $\beta(m,n) = \frac{\Gamma m\, \Gamma n}{\Gamma(m+n)}$ where m > 0, n > 0. (CO1) 6

3-c. Solve the differential equation by method of variation of parameter $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$. (CO 2) 6

3-d. Solve the differential equation $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ (CO 2) 6

3.e. Solve the differential equation $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (CO3) 6

3.f. Find inverse Laplace Transform of the function $f(s) = \log\frac{s(s+1)}{s^2+4}$. (CO 4) 6

3.g. (i) A sum of money becomes Rs.6690 after three years and Rs.10035 after 6 years on compound interest. Find the sum? 6

(ii) The difference between simple interest and compound interest on a sum for 2 years at 8%, when the interest is compounded annually Rs.16. If the interest was compounded half-yearly find the difference in two interests? (CO5)

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. Apply Dirichlet's integral to evaluate $\iiint x^2yz\, dx\, dy\, dz$, throughout the volume bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (CO1) 10

4-b. Evaluate $\iint_R (x-y)^4 e^{x+y}\, dx\, dy$ where R is the square with vertices at (1, 0), (2, 1), (1, 2) 10
where R is the square with vertices at (1, 0), (2, 1), (1, 2)
and (0, 1). (CO1)

5. Answer any <u>one</u> of the following:-

5-a. Solve $\frac{dx}{dt} + 2x - 3y = t,\ \frac{dy}{dt} - 3x + 2y = e^{2t}$. (CO 2) 10

5-b. Solve the following differential equation by changing the independent variable $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$. (CO 2) 10

6. Answer any <u>one</u> of the following:-

6-a. Solve the linear partial differential equation 10

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos(mx)\cos(ny) + 30(2x+y).$$ (CO3)

6-b. Solve : $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Solve the following differential equation by using Laplace transformation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = t\,e^{-t}$, Given that $x(0) = 1,\ x'(0) = 2$ (CO 4) 10

7-b. State convolution theorem and hence evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$. (CO 4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) What is Ram's present age, if after 8 years his age will be 10 times his age 10 years back? 10

(ii) The present age of Kabir is 50 years and that of his wife, Sarah is 40 years. How long ago was the ratio of their ages 3:2?

(iii) The sum of ages of 5 children born in a family at the intervals of 4 years each is 50 years. What is the age of the eldest child? (CO5)

8-b. (i) Two vessel contain milk and water in ratio 3:2 and 7:3. Find the ratio in which the contents of the two vessels have to be mixed to get a new mixture in which the ratio of milk and water is 2:1. 10

(ii) Alloy A contains 40% gold and 60% silver. Alloy B contains 35% gold and 40% silver and 25% copper. Alloy A and B are mixed in the ratio of 1:4. What is the ratio of gold and silver in the newly formed alloy? (CO5)