.

Printed Page:- 05 **Subject Code:- ACSBS0205**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2022-2023)

Subject: Linear Algebra

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. *If A be an n-rowed non singular matrix , X be an nx1 matrix , B be a nx1 null matrix, then the system of equation AX=B , has (CO1)* 1

(a) unique solution

(b) infinite solution

(c) more than two solutions

(d) none of these

1-b. The roots of the equation $\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0$ are (CO1) 1

(a) 0, 12 and 12

(b) 0, 12 and -12

(c) 0, 12 and 16

(d) none of these

.

1-c. A system of a non homogeneous linear equation AX=B having n unknown and rank of augmented matrix is r, always has a unique solution if (CO2) 1

(a) $r = n$

(b) $r < n$

(c) $r > n$

(d) none of these

1-d. If the system of equations ax + y = 3, x + 2y = 3, 3x + 4y =7 is consistent, then the value of a is given by (CO2) 1

(a) 2

(b) 1

(c) -1

(d) none of these

1-e. Which of the set of vectors are linearly dependent? (CO3) 1

(a) (1, 1, -1), 2, -3, 5), (-2, 1, 4)

(b) (1, -1, -1), (2, -3, 5), (-2, 1, 4)

(c) (1, 4, -1), 2, -2, 5), (-2, 1, 4)

(d) None of these

1-f. A subset W is called subspace of vector space V(F) for a, b ∈ F and α, β ∈ V, is satisfy -(CO3) 1

(a) $a\alpha - b\beta \in V$

(b) $a\alpha \times b\beta \in V$

(c) $a\alpha \div b\beta \in V$

(d) $a\alpha + b\beta \in V$

1-g. If A is skew-Hermitian matrix, then iA is (CO4) 1

(a) Skew-Hermitian matrix

(b) Hermitian matrix

(c) Symmetric matrix

(d) None of these

1-h. If λ is a characteristic root of the matrix A, then a characteristic root of the matrix A+kI is (CO4) 1

(a) λ

(b) $k + \lambda$

(c) $k - \lambda$

(d) None of these

1-i. In singular value decomposition method USV^T, where U is……… (CO5) 1

(a) Orthogonal

(b) Diagonal

(c) Transpose of orthogonal matrix

(d) None of these

1-j. PCA technique is used for…….. (CO5) 1

(a) Dimensionalty reduction

(b) Pattern recognition

(c) Orthogonalty reduction

(d) None of these

2. Attempt all parts:-

2.a. If $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$, then find the value of x,y, z and w. (CO1) 2

2.b. **Find the rank of the matrix** $A = \begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \end{bmatrix}$. (CO2) 2

2.c. Determine if the vectors {(1,-2,1), (2,1-1), (7,-4,1)} in R^3 are Linearly independent. (CO3) 2

2.d. Show that the matrix $A = \begin{bmatrix} 2 & 3-4i \\ 3+4i & 2 \end{bmatrix}$ is a Hermitian matrix. (CO4) 2

2.e. Define principal component analysis method. (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. **If** $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$. **Find the product AB and BA.** (CO1) 6

3-b. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$. (CO1) 6

3-c. Find the rank of a matrix reducing to normal form $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 1 & 2 & -8 \end{bmatrix}$. (CO2) 6

3-d. Test the consistency of system of equation 10y+3z=0,3x+3y+z=0,2x-3y-z=5,x+2y=4. (CO2) 6

3.e. Show that the vectors (2,1,4), (1,-1,2) and (3,1,-2) forms a basis of R^3.(CO3) 6

3.f. Show that the mapping $T: V_2(R) \rightarrow V_3(R)$ defined as $T(a,b) = (a+b, a-b, b)$ is 6 a linear transformation from $V_2(R)$ *into* $V_3(R)$. (CO4)

3.g. Calculate the covariance using PCA of the given data (CO5) 6

x:	2.5	0.5	2.2	1.9	3.1	2.3	2	1	1.5	1
y:	2.4	0.7	2.9	2.2	3.0	2.7	1.6	1.1	0.6	1.9

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. Compute Adj A and verify that A(Adj A)=(Adj A) A= A I, Given the matrix 10

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}.$$ (CO1)

4-b. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $n \in N$ (CO1) 10

and find the inverse of A.

5. Answer any <u>one</u> of the following:-

5-a. Determine the value of λ and μ so that the equations x+y+z=6, x+2y+3z=10, x+2y+λz=μ have 10

(i) no solution, (ii) a unique solution and (iii) infinite many solutions. (CO2)

5-b. Solve the following system of equation by LU decomposition method: (CO2) 10

2x+3y-z=5,3x+2y+z=10,x-5y+3z=0

6. Answer any <u>one</u> of the following:-

6-a. Show that the transformation T: $V_2(R) \rightarrow V_3(R)$ defined as 10

T(a, b)=(a+b, a-b, b) ∀ a,b ∊ R is linear. Find its null space, nullity, range and rank.(CO3)

6-b. Apply Gram-schmidt process to the vectors α_1=(1,0,1), α_2=(1,0,-1), α_3=(0,3,4) to obtain the orthonormal basis for $V_3(R)$.(CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. If $\lambda_1, \lambda_2, \ldots\lambda_n$ are the eigenvalues of a matrix A, prove the following: 10

(a) A^T has eigenvalues $\lambda_1, \lambda_2, \ldots\lambda_n$.

(b) If A is upper triangular, then its eigenvalues are exactly the main diagonal entries. (CO 4)

7-b. Show that A is skew- Hermitian and Unitary both, where $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$. (CO 4) 10

8. Answer any <u>one</u> of the following:-

8-a. Find the singular values of the $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and find the SVD decomposition of A. (CO5) 10

8-b. **Given the following data, use PCA to reduce the dimension from 2 to 1.**(CO5) 10

Feature	**Example 1**	**Example 2**	**Example 3**	**Example 4**
x:	4	8	13	7
y:	11	4	5	14