NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2023-2024)

Subject: Mathematical Foundation-II

Time: 3 Hours   **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A   20

1. Attempt all parts:-

1-a.   If $I = \int_0^1 \int_0^x x^2 \, dy dx$. Then   (CO 1)   1

   (i) $I = 0$

   (ii) $I = 1$

   (iii) $I = \frac{1}{2}$

   (iv) $I = \frac{1}{4}$

1-b.   Beta function $B(2,1) = a$. Then   (CO 1)   1

   (i) $a = \frac{1}{2}$

   (ii) $a = 2$

   (iii) $a = \frac{3}{2}$

   (iv) $a = 0$

1-c. The complementary function of $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = 0$ (CO 2)   1

      (i) $(c_1 x e^{2x} + c_2 e^{5x})$ (ii) $(c_1 e^{2x} + c_2 x e^{5x})$

      (iii) $c_1 e^{2x} + c_2 e^{5x}$ (iv) $(c_1 + c_2 x)e^{5x}$

1-d. The particular integral of $(D^2 + 5D + 6)y = e^x$ is   (CO 2)   1

      (i) $\frac{e^x}{6}$ (ii) $-\frac{e^x}{12}$

      (iii) $\frac{-e^x}{6}$ (iv) $\frac{e^x}{12}$

1-e. Solution of $yq - xp = z$ is   (CO 3)   1

      (i) $f(xy, y - z) = 0$ (ii) $f\left(\frac{x}{y}, y + z\right) = 0$

      (iii) $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ (iv) $f\left(xy, \frac{y}{z}\right) = 0$

1-f. PDE: $\frac{\partial u}{\partial t} + a\frac{\partial^2 u}{\partial x^2} = 0$, a is positive constant. The given PDE is (CO 3)   1

      (i)   Hyperbolic   (ii)   Elliptic

      (iii)   Parabolic   (iv)   None of these

1-g. If $L(f(t)) = F(s)$, then $L(e^{at} f(t)) = $ _____.   (CO 4)   1

      (i) $F(s - a)$ (ii) $F(s + a)$

      (iii) $F\left(\frac{s}{a}\right)$ (iv) $F(as)$

1-h. Find L(e^{-3t} Sinh 4t)   (CO 4)   1

      (i) $\frac{4}{(s+3)^2 - 16}$ (ii) $\frac{3}{(s+3)^2 + 16}$

      (iii) $\frac{4}{(s-3)^2 + 16}$ (iv) $\frac{3}{(s-3)^2 - 16}$

1-i. John is the son of Mary. Mary is the daughter of Robert. Robert is the father of Lisa. How is Lisa related to John?   (CO 5)   1

      (i)   Aunt   (ii)   Sister   (iii) Grand Mother   (iv)   Mother

1-j. A sum of Rs. 1500 amounts to Rs. 3000 in three years at compound interest. In how many years will the same amount become Rs. 6000? (CO 5)   1

(i) 3 years	(ii) 4 years	(iii) 5 years	(iv) 6 years

2. Attempt all parts:-

2.a. Evaluate $\int_0^1 x^4 \left(1 - \sqrt{x}\right)^5 dx$ (CO1) 2

2.b. Solve $(D^2 + 9) = Cos2x$ (CO2) 2

2.c. Solve $yzp + zxq = xy$ (CO3) 2

2.d. Find $L^{-1}\left(\dfrac{3s}{2s+9}\right)$ (CO4) 2

2.e. Two numbers are respectively 30% and 60% more than a third number. What is the 2
ratio of the two numbers? (CO 5)

SECTION – B 30

3. Answer any <u>five</u> of the following-

3-a. Apply Dirichlet's integral to find the volume of the solid surrounded by the surface 6
(CO1)

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1$$

3-b. Evaluate $\int\int_R (x + y)^2 \, dxdy$, where R is the parallelogram in the xy-plane with 6
vertices (1,0), (3,1), (2,2) , (0,1), using the transformation u = x + y and v = x - 2y
(CO1)

3-c. Solve by changing the independent variable (CO2) 6

$$Cosx\frac{d^2y}{dx^2} + Sinx\frac{dy}{dx} - 2y\,Cos^3x = 2\,Cos^5x$$

3-d. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = Sin(\log x)$ (CO2) 6

3-e. Solve $(y - z)p + (x - y)q = (z - x)$. (CO3) 6

3-f. If $L\{f(t)\} = F(s)$ then show that $L\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty F(s)\,ds$. (CO4) 6

3-g. Starting from a point P, Sachin walked 20 m towards South. He turned left and walked 6
30 m. He then turned left and walked 20m. He again turned left and walked 40 m and
reached a point Q. How far and in which direction is the point Q from the Point P?
(CO5)

4. Answer any <u>one</u> of the following-

4-a. Evaluate $\int_0^1 \int_y^1 \frac{x}{x^2+y^2} \, dx dy$ by changing the order of integration   (CO1)   10

4-b. Evaluate $\int \int_A (x^2 + y^2) \, dx dy$, throughout the area enclosed by the curves   10

$y = 4x, \quad x + y = 3, \quad y = 0 \quad \text{and } y = 2.$ (CO1)

5. Answer any <u>one</u> of the following-

5-a. Solve using variation of parameter $\frac{d^2 y}{dx^2} + y = cosec \, x.$ (CO2)   10

5-b. Solve simultaneous differential equations: $\frac{dx}{dt} = 3y - 2x$, $\frac{dy}{dt} = 3x - 2y.$ (CO2)   10

6. Answer any <u>one</u> of the following-

6-a. Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y) + e^{2x+y}$ (CO3)   10

6-b. Solve the partial differential equation :   (CO3)   10

$(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

7. Answer any <u>one</u> of the following-

7-a. State convolution theorem of Laplace transform and evaluate (CO4)   10

$$L^{-1}\left[\frac{1}{s^3(s^2 + 1)}\right]$$

7-b. Solve the initial value problem $\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 2y = 5 \sin t$, $y(0) = y'(0) = 0$ using   10
Laplace transform. (CO4)

8. Answer any <u>one</u> of the following-

8-a. (i) A sum of Rs. 10,000 is lent out at simple interest, and in 4 years, the total amount   10
becomes Rs. 12,800. What is the rate of interest per annum?

(ii) A borrower took a loan of Rs. 50,000 at a simple interest rate of 8% per annum.
He agreed to repay the loan in 4 equal annual installments. Each installment is paid at
the end of each year and includes both the principal and the interest accrued on the
outstanding amount. Calculate the amount of each installment and determine the total
interest paid over the entire period. (CO5)

8-b. John is currently three times as old as his son, Mark. Fifteen years ago, John was five   10
times as old as Mark was at that time. John's father, David, is twice as old as John.
Additionally, the sum of the ages of John, Mark, and David is currently 130 years.

(i) What are the current ages of John, Mark, and David?

(ii) In how many years will John be twice as old as Mark?

(iii) How old was John when David was the same age as John is now? (CO5)