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Subject Code:- BCSBS0205

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2023 - 2024)

Subject: Linear Algebra

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. If A is any square matrix, then which of the following is skew − symmetric? (CO1) 1

(a) $A+A^T$

(b) $A-A^T$

(c) AA^T

(d) none of these

1-b. If $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, then A^2+2A equals (CO1) 1

(a) $4A$

(b) $3A$

(c) $2A$

(d) none of these

1-c. The given system of vectors $X_1=(3,\ 1,\ -4)$, $X_2=(2,\ 2,\ -3)$, $X_3=(0,\ -4,\ 1)$ are (CO2) 1

(a) Linearly dependent

(b) Linearly independent

(c) consisitent

(d) none of these

1-d. A system of a non homogeneous linear equation AX=B having n unknown and rank of augmented matrix is r, always has a unique solution if (CO2) 1

(a) $r = n$

(b) $r < n$

(c) $r > n$

(d) none of these

1-e. Let T be a function from F^3 into F^3 then range of linear transformation T(a, b, c) = (a-b+2c, 2a+b, -a-2b+2c) is (CO3) 1

(a) (1, 1, 0), (1, 2, 3), (1, 2, -1)

(b) (1, 1, 1), (1, 2, 3), (1, -2, -1)

(c) (1, -1, 0), (1, 2, -3), (1, 2, -1)

(d) None of these

1-f. Two vectors α and β are orthogonal if and only if (CO3) 1

(a) $\|\alpha+\beta\|^2 = \|\alpha\|^2 - \|\beta\|^2$

(b) $\|\alpha+\beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$

(c) $\|\alpha+\beta\|^2 = \|\alpha\|^2 + \|\beta\|^2$

(d) $\|\alpha+\beta\|^2 = 2\|\alpha\|^2 + \|\beta\|^2$

1-g. If the eigen value of A is 2, then the eigen values of $A^3 + 2A^2 + A + I$ are (CO 4) 1

(a) 22

(b) 21

(c) 19

(d) None of these

1-h. If T_1 *and* T_2 *be a linear transformation* then which is correct? (CO 4) 1

(a) $(T_1 + T_2)(\alpha) = T_1(\alpha) - T_2(\alpha)$

(b) $(T_1 + T_2)(\alpha) = T_1(\alpha) + T_2(\alpha)$

(c) $(T_1 + T_2)(\alpha) = T_1(\alpha) \cdot T_2(\alpha)$

(d) None of these

1-i. PCA technique is used for........ (CO5) 1

(a) Dimensionalty reduction

(b) Pattern recognition

(c) Orthogonalty reduction

(d) None of these

1-j. In singular value decomposition method USV^T, where U is……… (CO5) 1

(a) Orthogonal

(b) Diagonal

(c) Transpose of orthogonal matrix

(d) None of these

2. Attempt all parts:-

2.a. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then find AB and BA. (CO1) 2

2.b. If $X = (1, 0, 0)$, $Y = (0, 1, 0)$ and $Z = (0, 0, 1)$, find $2X + 3Y - Z$. (CO2) 2

2.c. **If F is the field of real numbers, prove that the vectors (a_1, a_2) and (b_1, b_2) in $V_2(F)$ are linearly dependent iff $a_1b_2 - a_2b_1 = 0$.** (CO3) 2

2.d. Find the product of eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. (CO 4) 2

2.e. Define principal component analysis method. (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, verify that $(AB)' = B'A'$. (CO1) 6

3-b. Solve the following equations by Cramer's rule (CO1) 6
$x + 4y + 3z = 2$, $2x - 6y + 6z = -3$ *and* $5x - 2y + 3z = -5$.

3-c. Test the consistency of system of equation 10y+3z=0, 3x+3y+z=0, 2x-3y-z=5, x+2y=4. (CO2) 6

3-d. Show that the vectors $X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$ are linearly dependent and find the relation between them. (CO 2) 6

3.e. If u and v are vectors in a real inner product space. If II u II= II v II then u-v and u+v are orthogonal. (CO-3) 6

3.f. Find the eigen values of $3A^3 + 5A^2 - 6A + 2I$ where $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. (CO 4) 6

3.g. Given the following data, Using PCA find the covariance.(CO5) 6

x:	4	8	13	7
y:	11	4	5	14

SECTION C **50**

4. Answer any <u>one</u> of the following:-

4-a. Express $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ in to form of LU, where L is the lower traingular and U is the upper traingular matrix. (CO1) 10

4-b. Find the inverse of the matrix A by applying elementary transformations. (CO1) 10

$$A = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ -1 & 2 & 1 & -1 \\ 2 & -3 & -1 & 4 \end{bmatrix}.$$

5. Answer any <u>one</u> of the following:-

5-a. Reduce the matrix A to its normal form and hence find its rank where, (CO2) 10

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

5-b. Find the values of λ such that the following equations have unique solution $\lambda x + 2y - 2z - 1 = 0$, $4x + 2\lambda y - z - 2 = 0$, $6x + 6y + \lambda z - 3 = 0$ and use matrix method to solve these equation when $\lambda = 2$. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. Apply Gram- Schmidt process to transform the basis {(1,1,1), (0,1,1), (0,0,1)} into an orthonormal basis.(CO3) 10

6-b. If W_1 and W_2 are subspaces of the vector space $R^4(R)$ generated by S_1={(1,1,0,-1), (1,2,3,0), (2,3,3,-1)}, S_2={(1,2,2,-2), (2,3,2,-3), (1,3,4,-3)} respectively, Determine- (CO3) 10

(a) $\dim(W_1 + W_2)$

(b) $\dim(W_1 \cap W_2)$

7. Answer any <u>one</u> of the following:-

7-a. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$. (CO4) 10

7-b. Show that the mapping $T: V_3(R) \to V_2(R)$ defined as $T(a, b, c) = (a, b)$ is a linear transformation. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. Given the following data, use PCA to reduce the dimension from 2 to 1.(CO5) 10

Feature	Example 1	Example 2	Example 3	Example 4
x:	4	8	13	7
y:	11	4	5	14

8-b. Find a singular value decomposition of the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$. (CO5) 10