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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech.

SEM: III - CARRY OVER THEORY EXAMINATION - JUNE (2021 - 2022)

Subject: Discrete Structures

Time: 3 Hours

Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A

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1. Attempt all parts:-

- 1-a. Two sets are called disjoint if there _____ is the empty set. (CO1) 1
- (a) Union
 - (b) Difference
 - (c) Intersection
 - (d) Complement
- 1-b. For two sets C and D the set $(C - D) \cap D$ will be _____ (CO1) 1
- (a) C
 - (b) D
 - (c) Φ
 - (d) None of the mentioned
- 1-c. Let '*' be defined on the set N. Which of the following are both commutative and associative? (CO2) 1
- (a) $a*b=a+b$
 - (b) $a*b=a-b$
 - (c) $a*b=ab$
 - (d) $a*b=a$
- 1-d. Which of the following is not a type of binary operation? (CO2) 1
- (a) Transitive
 - (b) Commutative
 - (c) Associative
 - (d) Distributive
- 1-e. Which of the following relation is a partial order as well as an equivalence relation? (CO3) 1
- (a) equal to(=)
 - (b) less than(<)
 - (c) greater than(>)
 - (d) not equal to(!=)
- 1-f. Let G be the graph defined as the Hasse diagram for the \subseteq relation on the set $S\{1, 2, \dots, 18\}$. How many edges are there in G? (CO3) 1
- (a) 43722
 - (b) 2359296
 - (c) 6487535

(d) 131963

- 1-g. What is the value of x after this statement, assuming the initial value of x is 5? 'If x equals to one then $x=x+2$ else $x=0$. (CO4) 1
- (a) 1
(b) 3
(c) 0
- 1-h. Let P: If Sahil bowls, Saurabh hits a century.; Q: If Raju bowls, Sahil gets out on first ball. Now if P is true and Q is false then which of the following can be true? (CO4) 1
- (a) Raju bowled and Sahil got out on first ball
(b) Raju did not bowled
(c) Sahil bowled and Saurabh hits a century
(d) Sahil bowled and Saurabh got out
- 1-i. Another name for the directed graph is (CO5) 1
- (a) Direct graph
(b) Bigraph
(c) Dir-graph
(d) Digraph
- 1-j. The number of edges in a regular graph of degree 46 and 8 vertices is. (CO5) 1
- (a) 123
(b) 187
(c) 184
(d) 186

2. Attempt all parts:-

- 2.a. Define reflexive relation. (CO1) 2
- 2.b. If $(G, *)$ is a group and a is an element in G , such that $a * a = a$, then show that $a = e$, where e is identity element in G . (CO2) 2
- 2.c. Write any two properties of lattices. (CO3) 2
- 2.d. Show that $\neg(\neg p)$ and p are logically equivalent. (CO4) 2
- 2.e. Describe is complete graph?(CO5) 2

SECTION B

30

3. Answer any five of the following:-

- 3-a. If $A, B,$ and C are sets, using example show that: $A \cup B \cup C = A + B + C - A \cap B - A \cap C - B \cap C + A \cap B \cap C$. (CO1) 6
- 3-b. Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2, f(b) = 3,$ and $f(c) = 1$. Is f invertible, and if it is, what is its inverse? (CO1) 6
- 3-c. Let G be a group. Suppose that the number of elements in G of order 5 is 28. Determine the number of distinct subgroups of G of order 5. (CO2) 6
- 3-d. Let x, y be generators of a group G with relation: $xy^2=y^3x \dots(1), yx^2=x^3y \dots(2)$. Prove that G is the trivial group. (CO2) 6
- 3.e. Draw the Hasse diagram of (A, \leq) , where $A = \{3, 4, 12, 24, 48, 72\}$ and relation \leq be such that $a \leq b$ if a divides b . (CO3) 6
- 3.f. Use a truth table to verify the distributive law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$. (CO4) 6
- 3.g. (i) Give an example of bipartite graph. (ii) Graph having neither Euler nor Hamiltonian circuit. (CO5) 6

SECTION C

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4. Answer any one of the following:-

- 4-a. Explain the idea of Subset and Super set with example. State how a Null set can be a subset 10

- of Singleton set. (CO1)
- 4-b. Prove the Distributive law of algebraic structure for Union and Intersection. (CO1) 10
5. Answer any one of the following:-
- 5-a. Let G be a group and let H_1, H_2 be subgroups of G such that H_1 is not a subset of H_2 and H_2 is not a subset of H_1 . (a) Prove that the union $H_1 \cup H_2$ is never a subgroup in G . (b) Prove that a group cannot be written as the union of two proper subgroups. (CO2) 10
- 5-b. Let G be an abelian group. Let a and b be elements in G of order m and n , respectively. Prove that there exists an element c in G such that the order of c is the least common multiple of m and n . Also determine whether the statement is true if G is a non-abelian group. (CO2) 10
6. Answer any one of the following:-
- 6-a. for each of following expression . Find the minimum sum of product using k-map. 1). $A'B'C' + AB'C' + A'BC' + ABC'$ (CO3) . 10
- 6-b. Answer these questions for the poset($\{3, 5, 9, 15, 24, 45\}, \mid$). (CO3) 10
 i. Find the minimal elements.
 ii. Is there a greatest element?
 iii. Is there a least element?
 iv. Find all upper bounds of $\{3, 5\}$.
7. Answer any one of the following:-
- 7-a. Use proof by contradiction to prove that the sum of an irrational number and a rational number is irrational. (CO4) 10
- 7-b. Show that each of these conditional statements is a tautology by using truth tables. (CO4) 10
 a) $(p \wedge q) \rightarrow p$,
 b) $p \rightarrow (p \vee q)$,
 c) $\neg p \rightarrow (p \rightarrow q)$,
 d) $(p \wedge q) \rightarrow (p \rightarrow q)$,
 e) $\neg(p \rightarrow q) \rightarrow p$, f) $\neg(p \rightarrow q) \rightarrow \neg q$
8. Answer any one of the following:-
- 8-a. Define planar graph. Prove that for any connected planar graph, $v - e + r = 2$ Where v, e, r is the number of vertices, edges, and regions of the graph respectively. (CO5) 10
- 8-b. (a) Suppose a graph G contains two distinct paths from vertex u to a vertex v . Show that G has a cycle. (b) Find the number of connected graph with 4 vertices. Also draw the graph. (CO5) 10